An Information-Based Theory of Time-Varying Liquidity

Brett Green
UC Berkeley, Haas School of Business

joint with

Brendan Daley
Duke University, Fuqua School of Business

June 2013

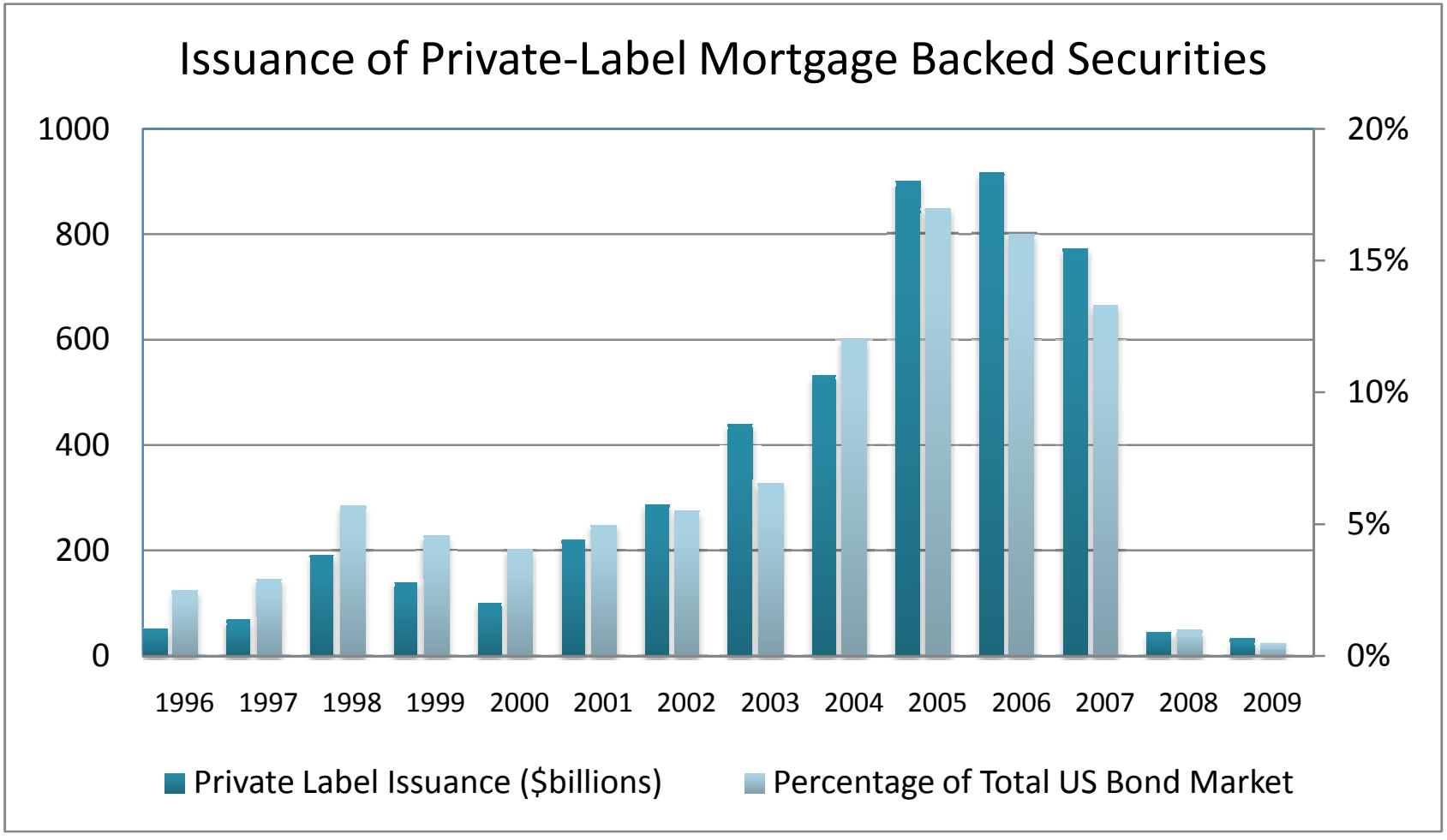
Motivation

Markets are susceptible to periods of illiqudity. Recent examples include:

- Real estate (Clayton, MacKinnon, and Peng, 2008)
- Mortgage backed securities (Gorton, 2009; Acharya and Schnabl, 2010; Dwyer and Tkac, 2009)
- Repo markets (Gorton and Metrick, 2012)
- Structured credit (Brunnermeier, 2009)
- Commercial paper (Anderson and Gascon, 2009)
- Money market funds (Krishnamurthy, Nagel, and Orlov, 2012)

We propose an information-based theory to explain such episodes and explore the impact on prices and volatility.

Motivation



(Source: SIFMA)

Key Features of the Model

The model takes place in a competitive dynamic economy with fully-rational, risk-neutral agents who share a common-prior.

The three key features are:

1. *Asymmetric Information:* the asset owners are privately informed about future cash flows.
2. *News:* information about cash flows is gradually and stochastically revealed to the market.
3. *Shocks:* agents are subject to idiosynchratic shocks. Upon arrival, agent is *not forced* to sell, but is more eager to do so.

Preview of Main Results

1. *Time Varying Liquidity*.
   - Equilibrium involves periods of full, partial, and zero liquidity.
2. *Illiquidity Discount*.
   - Illiquidity leads to an endogenous liquidation cost.
   - Buyers anticipate these costs driving prices below fundamentals.
3. *Excess Volatility*. Bad news gets compounded.
   - Negative signal about fundamentals.
   - Negative signal about future liquidity.
4. *(Efficient) Fire Sales*. Due to informational externalities.
   - A trade by one owner can reveal information,
   - which facilitates trade by other owners.

Related Literature

We build on Daley and Green, 2012:

- Single privately informed seller; competetive buyers.
- News revealed gradually.
- Trade occurs only once.

By incorporating two features:

1. Idiosyncratic (financial/credit/preference) shocks.
2. Multiple shares and multiple informed owners.

Focus on the first, consider the second in an extension.

Some Related Literature

Asymmetric Information and Liquidity

- Lucas and McDonald (1990), Korajczyk, Lucas, and McDonald (1992), N. B. Gârleanu and Pedersen (2003), Eisfeldt (2004), and Vayanos and J. Wang (2012)...

Transaction costs based theories of illiquidity

- Amihud and Mendelson, 1986; Constantinides, 1986; Vayanos, 1998; Vayanos, 2004; Lo, Mamaysky, and J. Wang, 2004; Acharya and Pedersen, 2005...

Search based theories of illiquidity

- Duffie, N. Gârleanu, and Pedersen (2005), Duffie, N. Gârleanu, and Pedersen (2007), Vayanos (1998), Vayanos and T. Wang (2007), and Vayanos and Weill (2008)...

The Model

Agents:

- Initial owner, A_0
- Owner at time t, A_t
- Many potential buyers (the "market")
  - Buyers not modeled directly, though it is possible to do so.

Preferences:

- All agents are risk-neutral and
- Agents discount future cash flows at rate r

The Model

The Asset:

- Single (indivisible) asset of type $\theta \in \{L, H\}$
- Nature chooses θ with $P_0 = \mathbb{P}(\theta = H)$
- The current owner knows θ and accrues (stochastic) cash flow with mean v_θ
- High-value asset pays more: $v_H > v_L$
- Let $V_\theta \equiv \int_0^\infty e^{-rt} v_\theta dt$

Idiosyncratic Shocks

All agents in the economy face idiosyncratic shocks:

- Publicly observable shocks arrives according to Poisson process with arrival rate λ.
- Arrival of shock introduces a holding cost c_θ.
  - v_θ if she has not been hit by a shock (*holder*)
  - $k_\theta \equiv v_\theta - c_\theta$ if she has been hit by a shock (*seller*)
- Generates gains from repeated trade, but does not *force* the owner to sell.
  - $k_H > v_L$ so that shocks are not overly punitive.
  - Preserves strategic considerations.

News Arrival

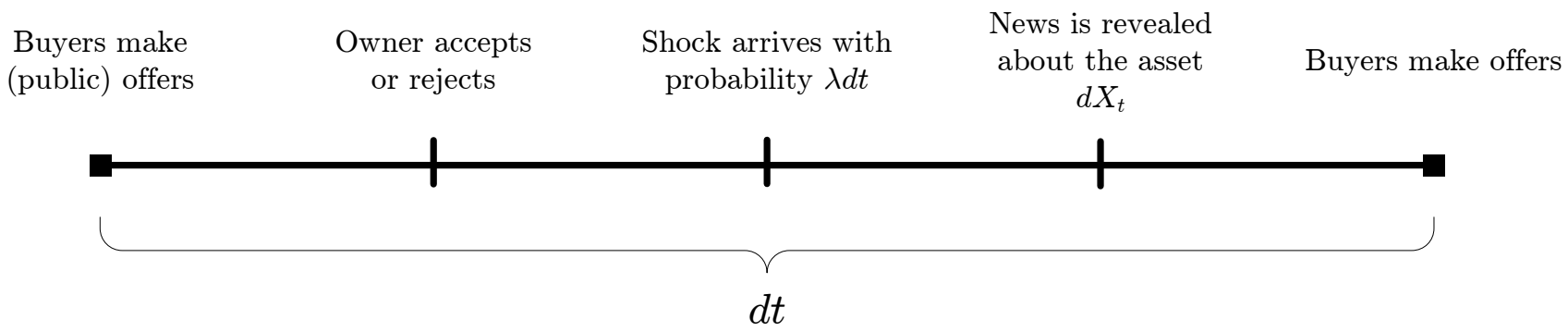
- Brownian motion drives the arrival of *news*.
- A publicly observable *score* process (X_t) evolves according to:

$$dX_t = \mu_\theta dt + \sigma dB_t$$

  where $\mu_H \geq \mu_L$
- The *quality* (or *speed*) of the news is measured by the signal-to-noise ratio: $\phi \equiv \frac{\mu_H - \mu_L}{\sigma}$
- One possible interpretation:
  - News=cashflows: $\mu_\theta = v_\theta$

Timing

- Infinite-horizon, continuous-time setting
- Trading mechanism: at every t
  - Buyers make offers.
  - Owner decides which offer to accept (if any).
  - Alternative: Seller post price.
- Owners that trade exit the economy.
- News and shocks are realized and repeat.



First best benchmark: Shocked owners (sellers) trade immediately.

- Informational friction inhibits efficiency.

Market Beliefs and Buyers' Strategy

Buyers begin with common prior: $P_0 = \mathbb{P}_{t=0}(\theta = H)$

- At time t, buyers know:
  - (i) The path of news arrival, shocks and offers up to time t
  - (ii) All times prior to t (if any) at which the asset has traded

Buyers' Strategy

- The buyer's strategy is a bid process W
  - $W_t(\omega)$ is the (maximal) bid made in the history (t, ω)

Equilibrium Beliefs

- Let P denote the equilibrium belief process held by buyers:

$$P_t(\omega) = \mathbb{P}(\theta = H | \mathcal{F}_t^B)$$

- Define $Z = \ln\left(\frac{P}{1-P}\right)$: "beliefs" in z-space

Skip Ahead

Owner's Strategy and Sequential Rationality

Owner's Strategy

- The strategy of an owner is a stopping rule τ.

Definition (Sequential Rationality)

Given W, an owner's strategy is sequentially rational if for all histories, it solves:

$$\sup_{\tau} E_t^{\theta}\left[\int_t^{\tau} e^{-rs}(v_\theta - I_s c_\theta)ds + e^{-r(\tau-t)}W_\tau | \mathcal{F}_t^S\right] \qquad (SP_\theta)$$

Equilibrium Concept

Definition

An equilibrium is a triple (τ, W, Z):

- Given W, the owner's strategy is sequentially rational.
- Given τ and Z, W is such that
  - Buyers earn zero profit.
  - No (profitable) deals exist.
- Market beliefs, Z, are consistent with Bayes rule whenever possible.

Equilibrium Beliefs

In equilibrium, the market beliefs evolves based on news as well as:

- The owner's equilibrium strategy and
- Previous trades (or lack thereof)

That is,

$$dZ_t = \underbrace{d\hat{Z}_t}_{\text{updating based only on news}} + \underbrace{dQ_t}_{\text{updating based on trades}}$$

Where dQ_t is the information in whether trade occurred at time t.

For example, suppose trade does not occur at time t:

- If strategies call for trade with probability zero: $dQ_t = 0$
- If strategies call for a low type to trade with positive probability and a high type to trade with probability zero: $dQ_t > 0$

Equilibrium Description

Equilibrium is stationary w.r.t. (z, i); any history such that:

- Market beliefs are z
- The owner's status is i
  - $i = 1$ indicates seller (positive holding cost)
  - $i = 0$ indicates holder (zero holding cost)

Equilibrium Characterization

Theorem

There exists an equilbrium. It is characterized by

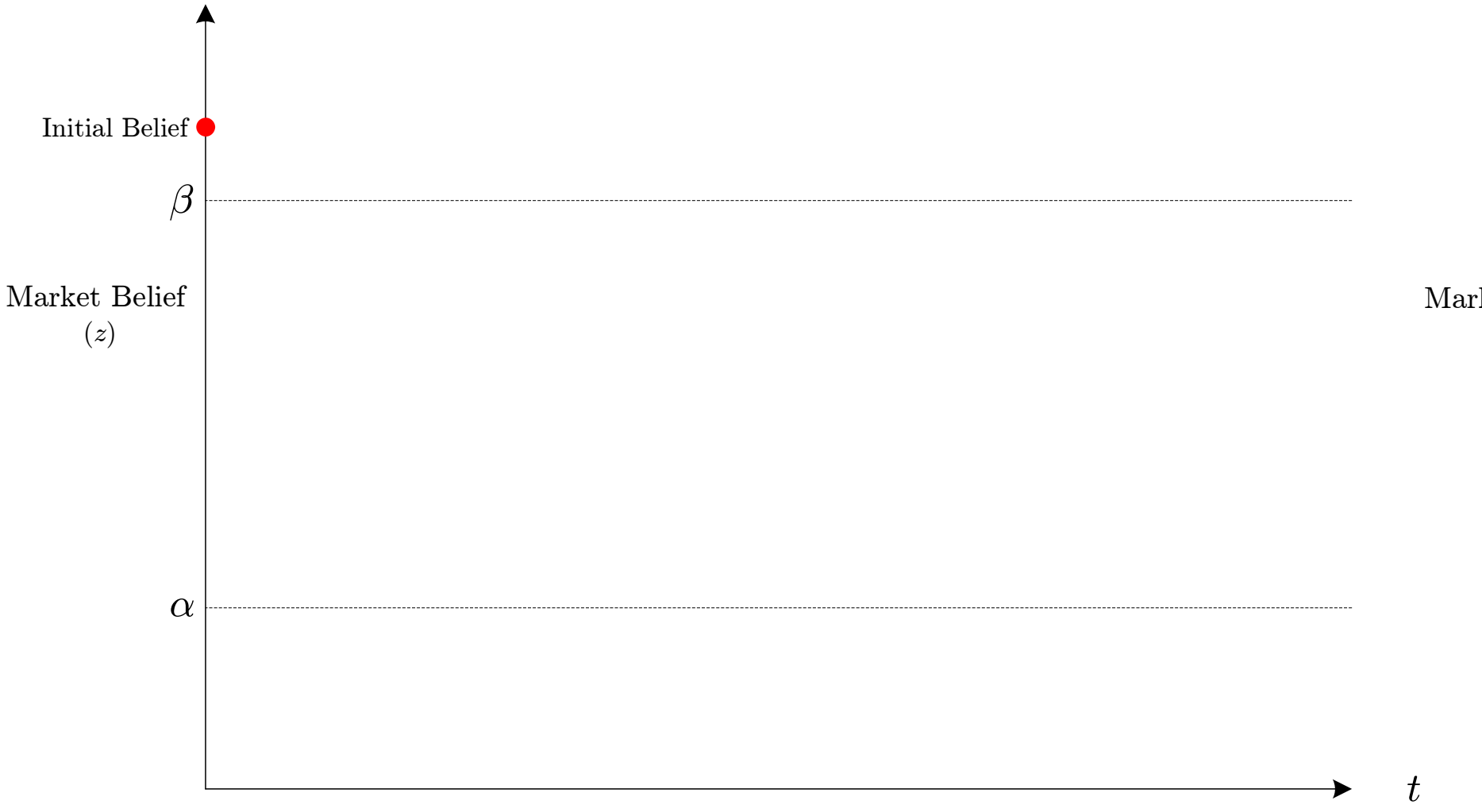
- $(\alpha, \beta) \in \mathbb{R}^2$ *and* $B(z) : \mathbb{R} \to \mathbb{R}$

and the following three regions when owner is a seller $i = 1$,

1. *If* $z \geq \beta$*: the market is* **fully liquid**.
   - *Bid is* $B(z)$ *and both types accept w.p.1.*
2. *If* $z \leq \alpha$*: the market is* **partially liquid**.
   - *Bid is* V_L*. High type rejects. Low type mixes.*
3. *If* $z \in (\alpha, \beta)$*: the market is* **fully illiquid**.
   - *Bid is rejected w.p.1.*

When the owner is a holder $(i = 0)$*, it is common knowledge there are no gains from trade and trade does not occur.*

Sample Path of Play



Proof by Construction

Step 1: Take B and (α, β) as given. Construct seller value functions F_L, F_H through ODEs and two sets of boundary conditions:

- Physical conditions (e.g., value matching).
- Necessary local Optimality conditions (e.g., smooth pasting).

Step 2: Taking F_L, F_H as given. Construct holder value functions G_L, G_H through ODEs and boundary conditions.

Step 3: Taking G_L, G_H as given, a buyers value is the expected value to a holder given both types sell:

$$B(z) = E[G_\theta(z)|z]$$

Step 4: Show there exists a fixed point of the system in Steps 1-3.

Step 5: Verify necessary optimality conditions are sufficient.

Intuition

Take B as given:

1. H can always get $B(z)$ if she wants it. For $z < \beta$, she does better by not exercising the option.
2. For high enough z, H has little to gain by waiting for good news so she exercises.
3. L can always get V_L if he wants it. But for $z \in (\alpha, \beta)$, he does better to mimic H.
4. L's prospects of reaching β decrease with z. At $z = \alpha$, he is just indifferent $\Longrightarrow$ willing to mix.

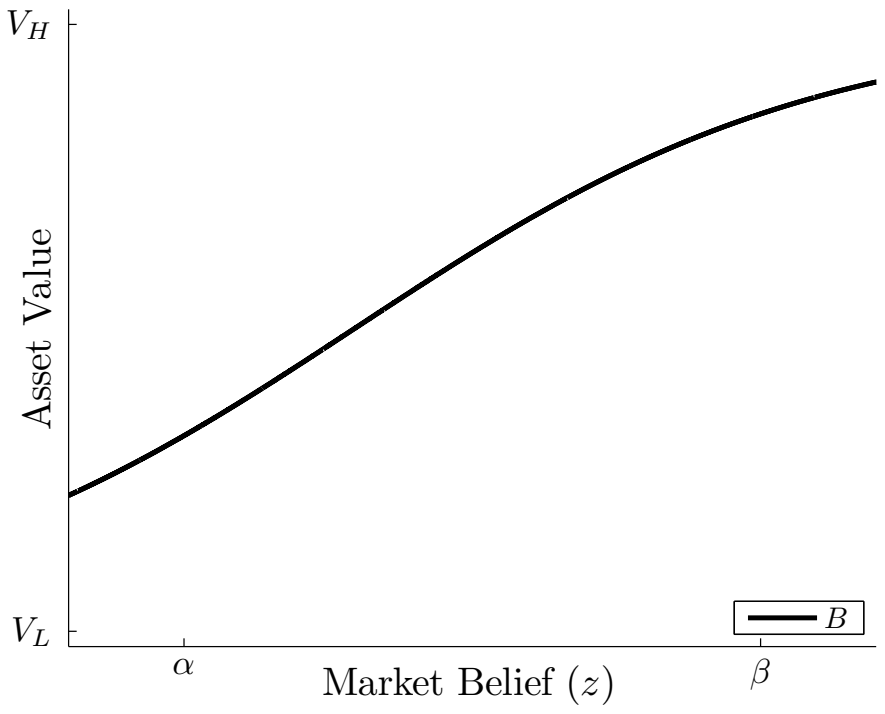


Figure: Constructing F_θ from B

Buyer and Holder Values

- Of course, B, depends on a holder's value, G_θ, which in turn depends on a seller's value, F_θ.
  - Fixed point characterizes this interdependence.
- Useful to compare to two benchmark cases.
  1. Benchmark 1: No private information. All agents symmetrically uninformed about θ.
  2. Benchmark 2: No shocks. Set $\lambda = 0$.

Benchmark 1: No Private Information

Suppose owners and buyers are commonly uninformed about θ.

Then, upon arrival of a shock:

(i) A seller has no reason to delay trade.

(ii) Given any market belief z, buyers are willing to pay the expected **fundamental value** of the asset.

$$B(z) = \Psi(z) \equiv \mathbb{E}\left[V_\theta | z\right]$$

(iii) Therefore, the market is fully liquid for all z and trade occurs immediately at $\Psi(z)$.

Benchmark 2: No Financial Shocks

When $\lambda = 0$, there is a unique three-region equilibrium.

(i) Because holders never face the need to resell,

$$G_\theta(z) = v_\theta.$$

(ii) Buyers still face potential adverse selection, but need not worry about future liquidation costs. Their (unconditional) value is:

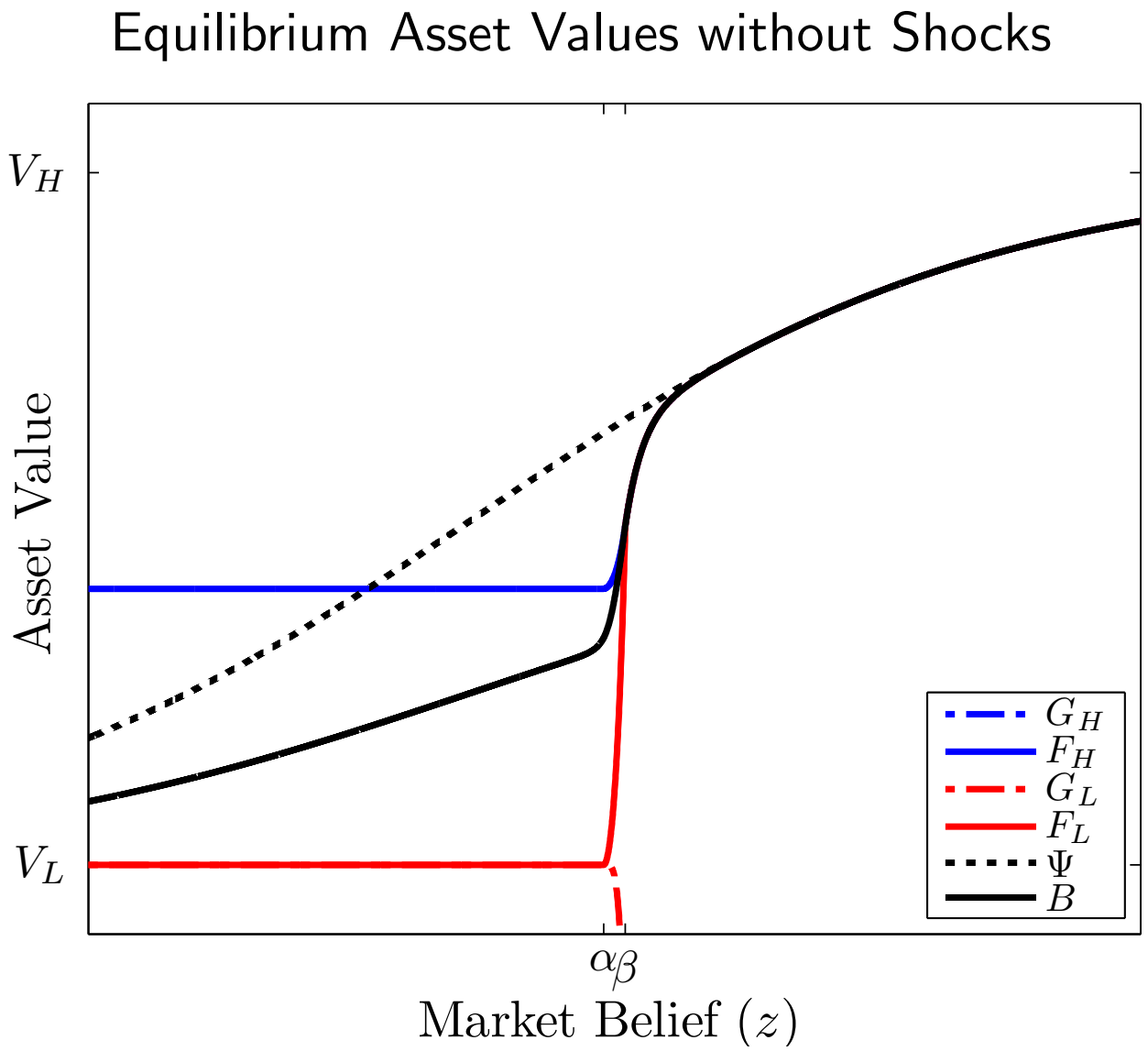
$$B(z) = \Psi(z)$$

(iii) The asset trades only once and

$$\text{Price} = \text{fundamental value}$$

.

Benchmark 2: Equilibrium Value Functions



The Illiquidity Discount

Two properties from benchmarks:

1. $B(z) = \Psi(z)$, and
2. Asset always trades at fundamental value.

With both private information and shocks, these no longer hold.

- Holder face the potential of costly future liquidation.

$$G_\theta < \frac{v_\theta}{r}$$

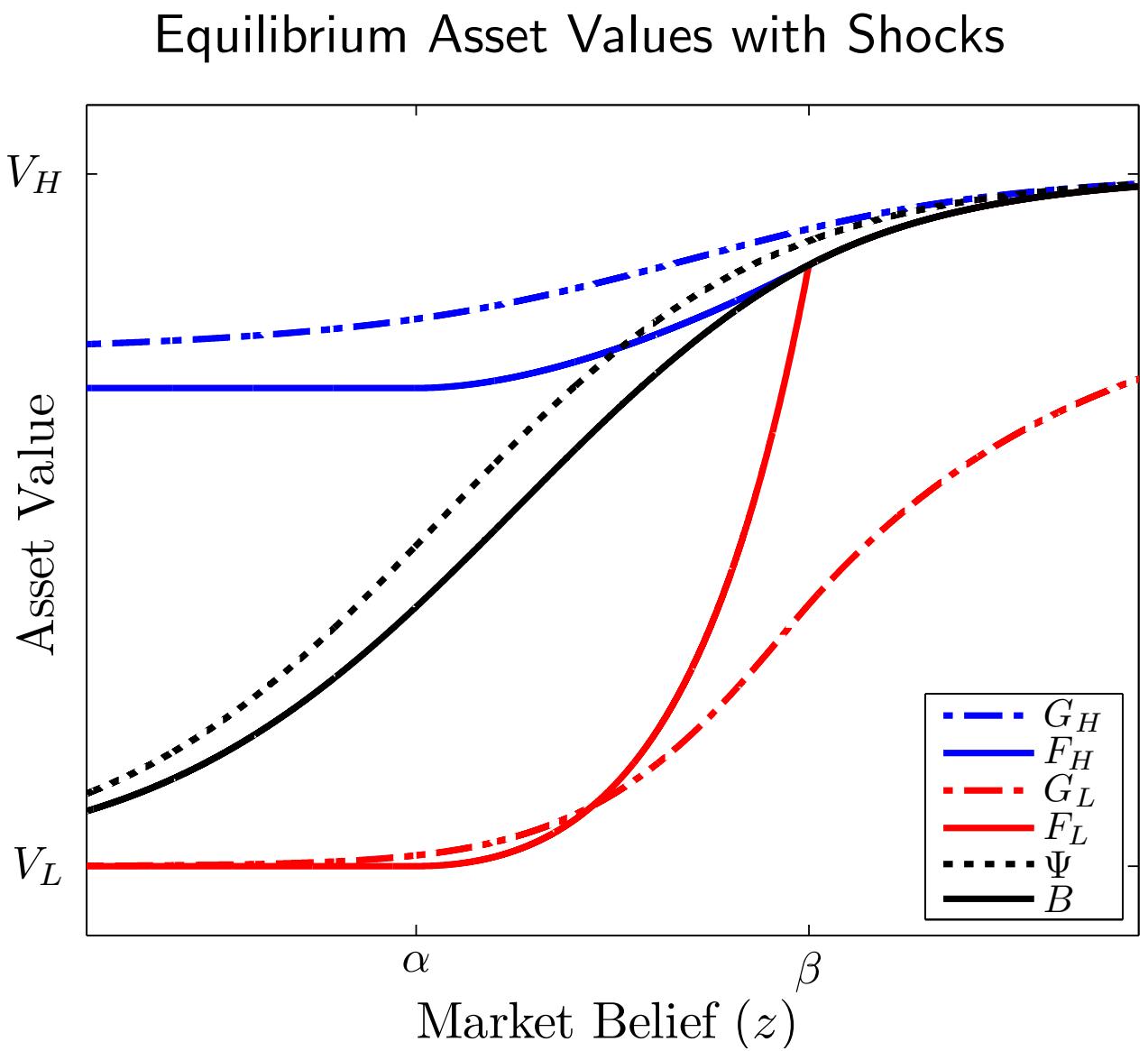
- As a result, buyers shade bids below fundamentals.

Proposition (The Illiquidity Discount)

When the market is fully liquid, trade takes place at a price strictly below the fundamental value.

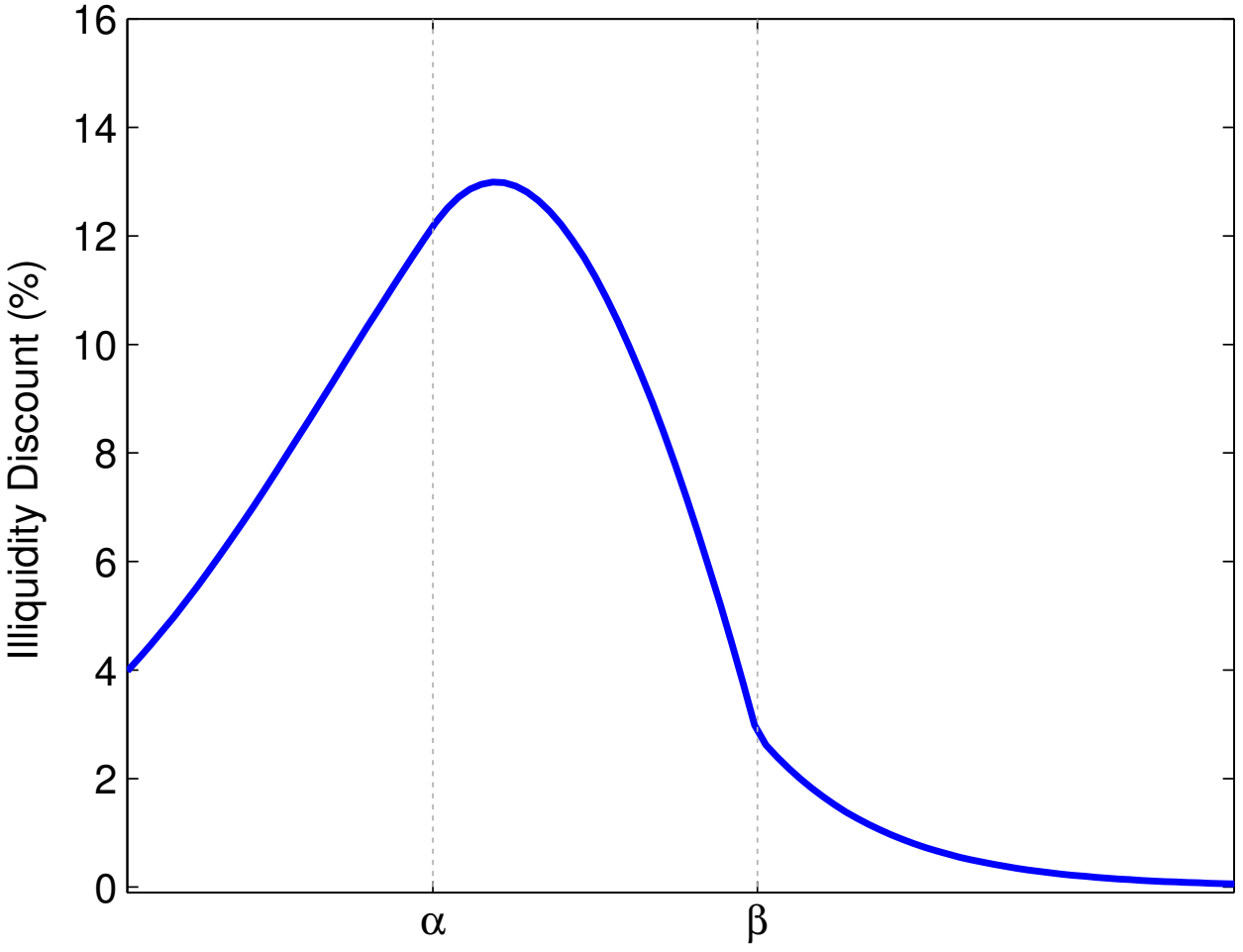
$$B(z) = \Psi(z) - \underbrace{\delta(z)}_{\text{illiquidity discount}}$$

Equilibrium Value Functions



The Illiquidity Discount

The illiquidity discount as measured by $\frac{\Psi - B}{\Psi}$.



Excess Volatility

Proposition

In fully liquid markets, the volatility of the equilibrium price process is strictly greater than the fundamental volatility. That is

$$B'(z) > \Psi'(z).$$

Intuition: Starting from any $z \geq \beta$, bad news has two affects.

1. Reduces traders' expectations about fundamentals.
2. Increases likelihood of future illiquidity.

The effect of bad news gets amplified, generating additional volatility.

Market Efficiency

Two ways to measure efficiency:

1. Trade Volume—frequency with which asset is (efficiently) transferred.
2. Value Loss—fraction of total value realized.

$$\mathcal{L}^F \equiv \frac{\Psi(z) - E[F_\theta(z)|z]}{\Psi(z)}, \quad \text{or} \quad \mathcal{L}^G \equiv \frac{\Psi(z) - E[G_\theta(z)|z]}{\Psi(z)}$$

Note: Focus on value loss measure here. Results similar for volume.

Efficiency: Value Loss

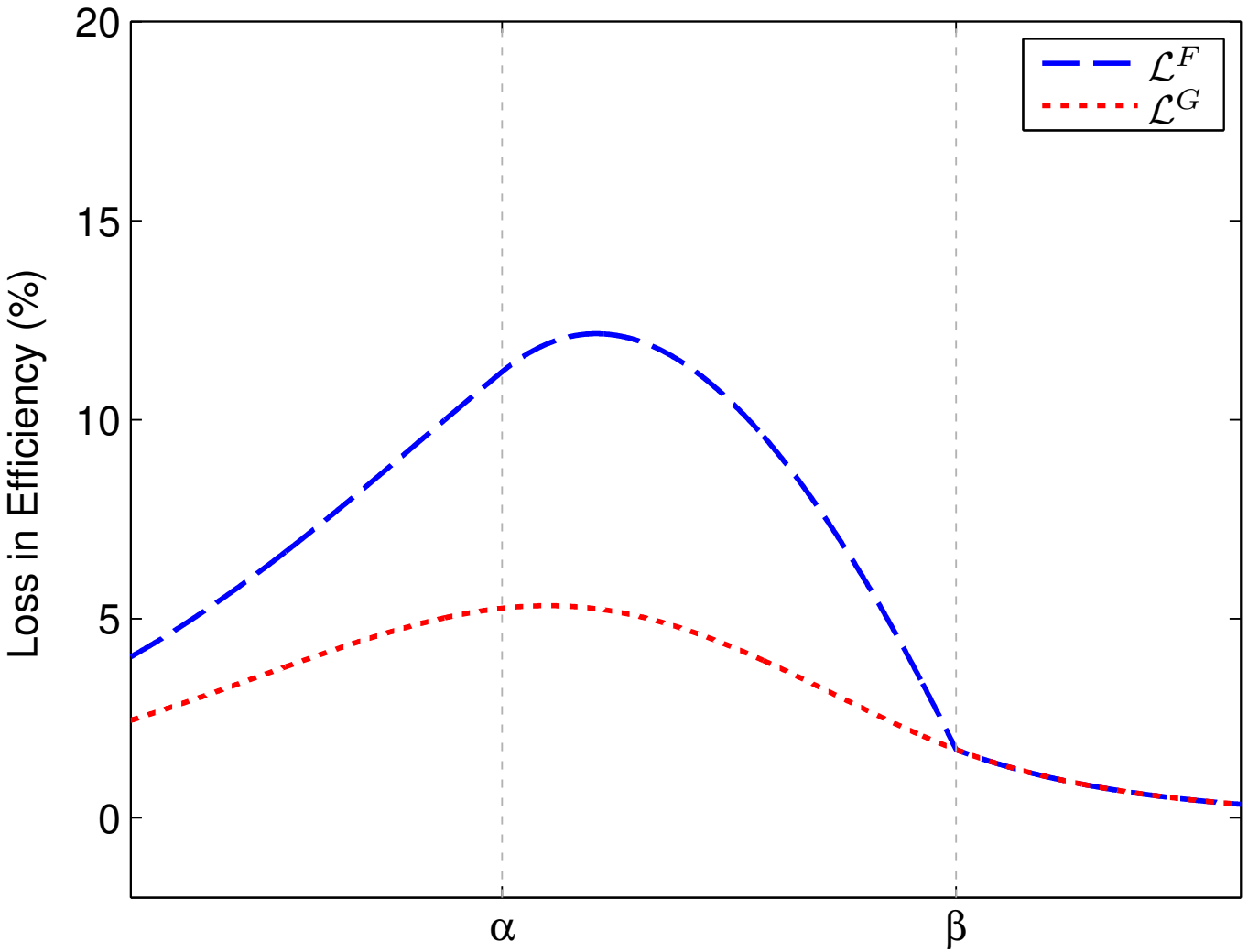


Figure: Efficiency as it depends on z, i.

Efficiency and News Quality

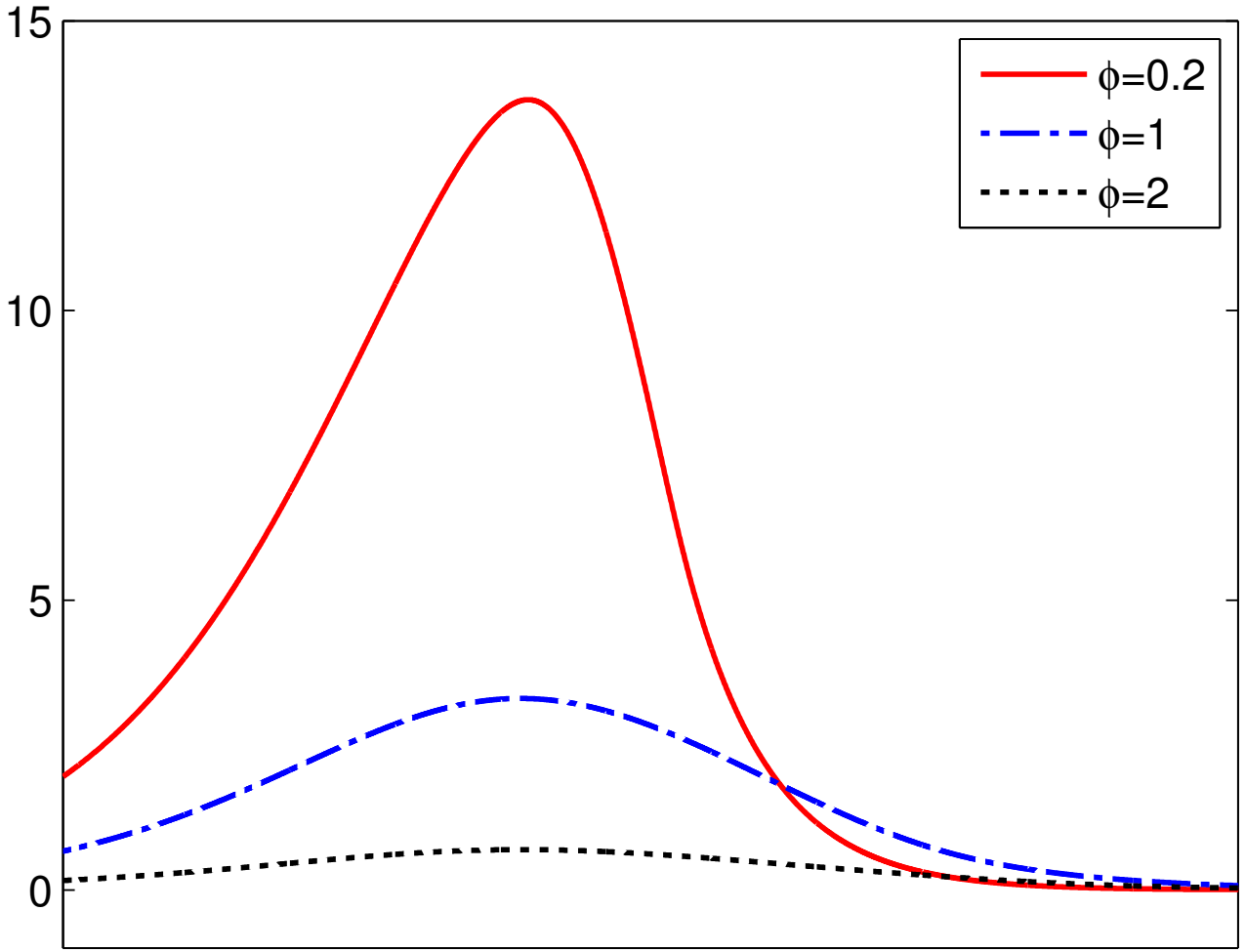


Figure: Efficiency may increase or decrease with news quality depending on the initial state.

Efficiency and Arrival of Shocks

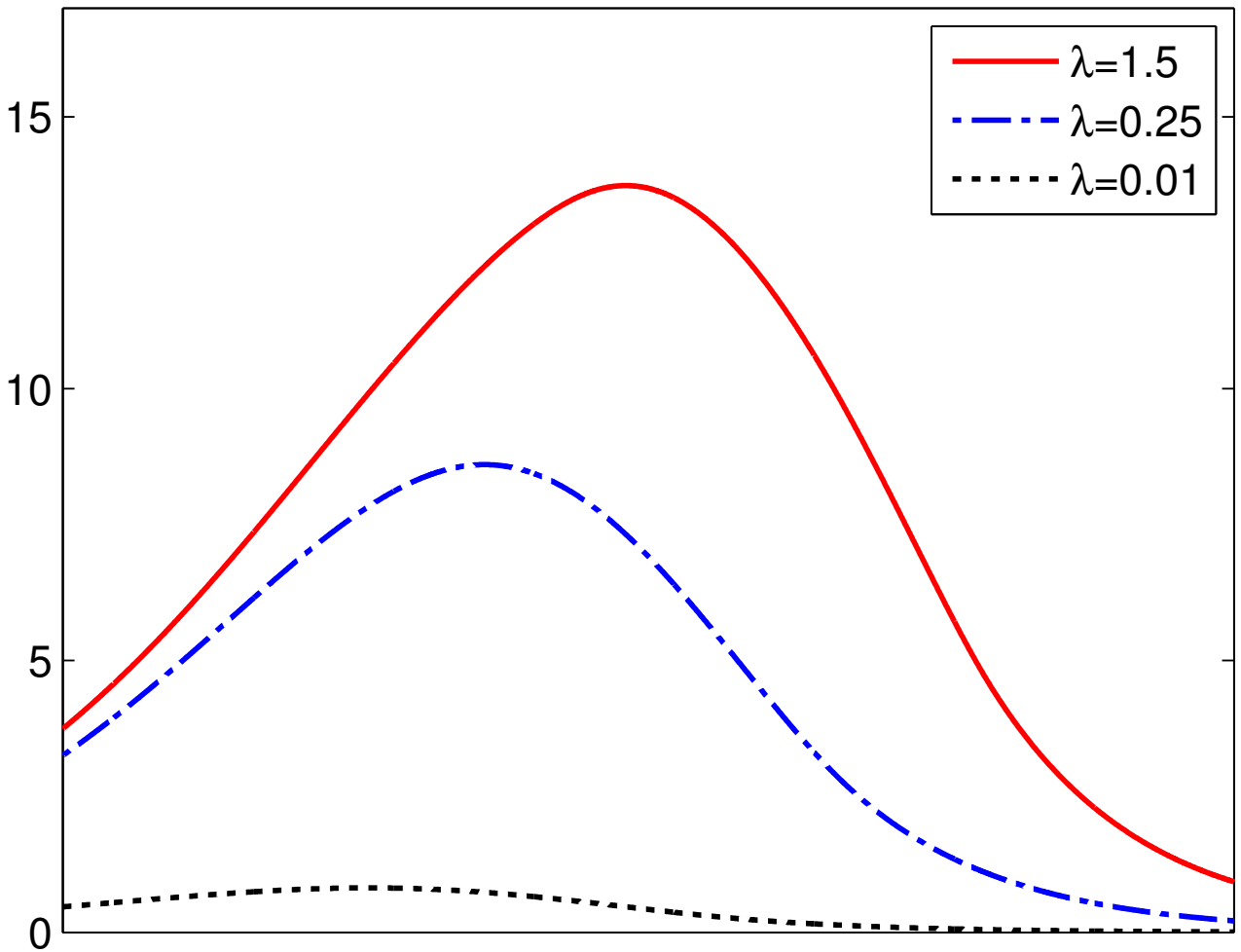


Figure: Efficiency decreases with the arrival rate of shocks—costly liquidation occurs more frequently.

Efficiency and Holding Costs

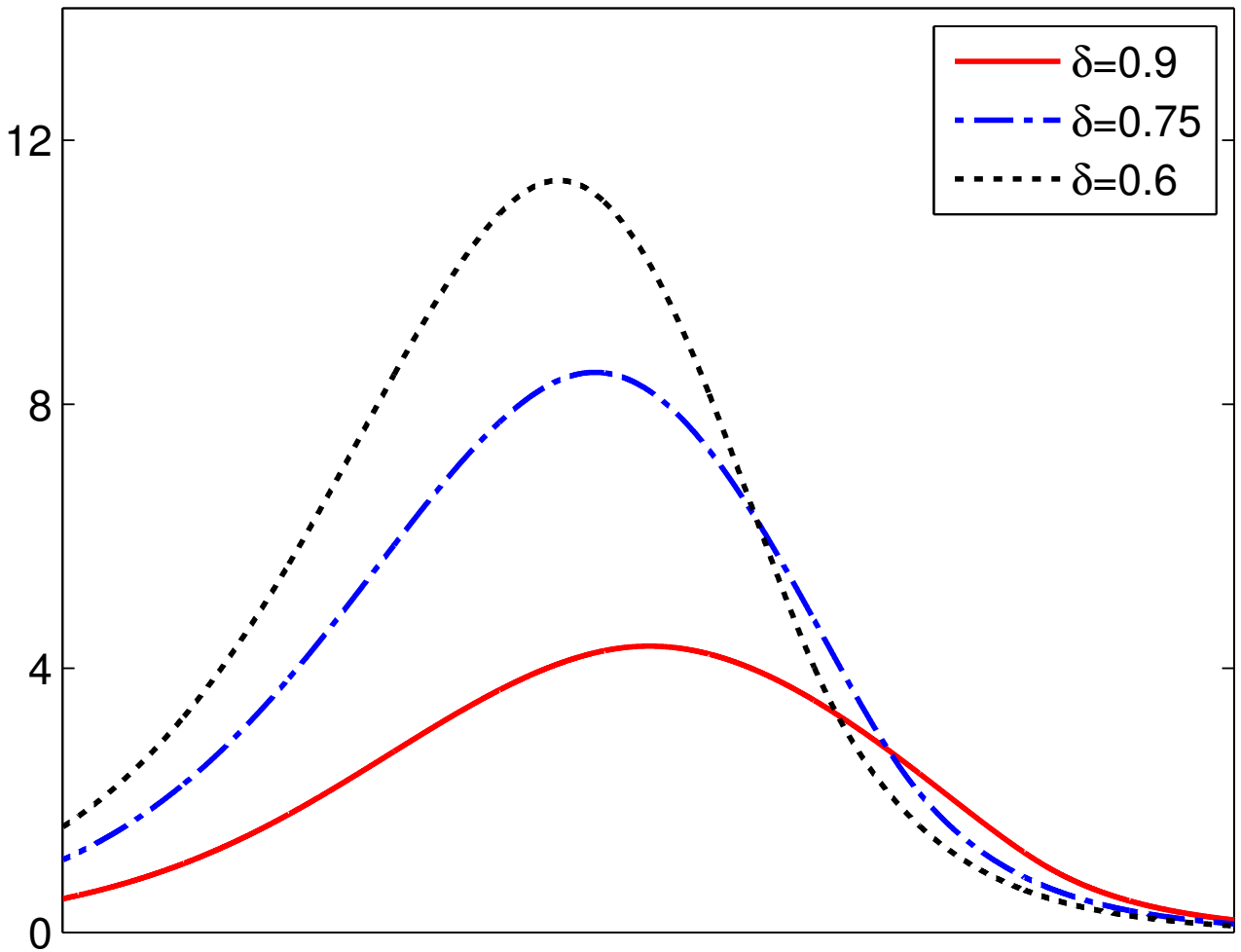


Figure: Efficiency can increase or decrease with the severity of the shock. Lower δ corresponds to higher holding costs ($\delta \equiv \frac{v_\theta - c_\theta}{v_\theta}$).

Markets with Multiple Shares

- We extend the model to a setting with N identical shares.
  - Each agent can own at most one share.
- Possible Interpretations
  - Dispersion of (informed) ownership and
  - Transparency of trades
- Purpose?
  - Application to a broader range of markets
  - The role information externalities
  - Robustness

Information Externalities and Fire Sales

Two interesting results from the N-share model:

1. Fire Sales
   - One seller's trade at $z = \alpha$, reveals $\theta = L$.
   - Other sellers have no (further) reason to delay.
   - Holders sell immediately upon arrival of shock.

2. Implications for Efficiency
   - The presence of other informed sellers leads to faster information revelation.
   - This affects equilibrium asset values.
   - Improves overall market efficiency (in contrast to ϕ).

Final Remarks

Presented a theory of time-varying liquidity based on:

- Private information
- News Revelation
- Idiosyncratic Shocks

Model also generates

- An illiquidity discount
- Excess volatility
- Fire sales

Discussed implications for market efficiency.