Signal or noise? Uncertainty and learning about whether other traders are informed

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Our framework: Uninformed investors are **uncertain** about other traders, and must **learn** about them

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- Rational investors are uncertain about whether others are trading on informative signals or noise
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- Stochastic, predictable expected returns and volatility
- Volatility clustering and the "leverage" effect
- Disagreement-return relation is non-monotonic and time-varying

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Underlying Mechanism:

(i) *Uncertainty about others* leads to non-linearity in prices,
(ii) *Learning about others* generates persistence

Related Literature

- Uncertainty about others
  - Participation / proportion of informed: Cao, Coval and Hirshleifer (2002), Romer (1993), Lee (1998), Avery and Zemsky (1998), Gao, Song and Wang (2012)
  - Existence / Precision of informed: Gervais (1997), Li (2013), Back, Crotty and Li (2014, 2015)
  - Risk Aversion: Easley, O'Hara and Yang (2013)
- Stochastic volatility and expected returns through learning
  - Regime switching models: David (1997), Veronesi (1999), Timmermann (2001)
  - Stochastic volatility of noise trading: Fos and Collin-Dufresne (2012)
- Non-linearity in prices
  - Ausubel (1990), Foster and Viswanathan (1993), Rochet and Vila (1994), DeMarzo and Skiadas (1998), Barlevy and Veronesi (2000), Spiegel and Subrahmanyam (2000), Breon-Drish (2010), Albagli, Hellwig, and Tsyvinski (2011)
- Investors "agree to disagree" but update beliefs using prices
  - Banerjee, Kaniel and Kremer (2009)

Benchmark Model: Payoffs and Preferences

Two date, two securities

- Risk-free asset with return normalized to $R = 1 + r$
- Risky asset has price P and pays dividends

$$D = \mu + d, \quad \text{where } d \sim \mathcal{N}(0, \sigma^2)$$

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Aggregate supply of the risky asset is *constant*:

$$\sum_i x_i = Z$$

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   Informed ($\theta = I$)**:** (e.g., institutions) Receive informative signals

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Informed ($\theta = I$)**:** (e.g., institutions) Receive informative signals

$$S_I = d + \varepsilon_I, \quad \varepsilon_I \sim \mathcal{N}(0, \sigma_e^2)$$

Noise / Sentiment ($\theta = N$)**:** (e.g., retail) Receive uninformative signals

$$S_N = u + \varepsilon_N, \quad \varepsilon_N \sim \mathcal{N}(0, \sigma_e^2), u \sim \mathcal{N}(0, \sigma^2),$$

which they incorrectly believe to be informative about dividends.

- Empirically relevant: over-confidence or differences of opinion
- Unconditional distribution of S_N and S_I are identical

Uncertainty about other investors

Key Feature: U investors are **uncertain** about who they face

- At any date t, either N or I investors are present (but not both)
- Denote the type of trader at date t by $\theta \in \{I, N\}$.
- Denote the likelihood of others being informed by $\pi = \Pr(\theta = I | \mathcal{I}_U)$

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Model nests rational expectations (RE) and differences of opinions (DO)

- When $\pi = 1$, U and θ investors have common priors (RE)
- When $\pi = 0$, U and θ investors agree to disagree (DO)

Characterizing Equilibria

Following Kreps (1977), we assume investors can observe residual supply

- Non-existence when U investors only observe price

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Important: Signal-revealing $\neq$ fully informative

- U investors are uncertain about fundamentals since they don't know whether θ is informed!

Learning about dividends

Investor θ's beliefs about d are:

$$\mathbb{E}_\theta [d] = \lambda S_\theta \quad \text{and} \quad \text{var}_\theta [d] = \sigma^2(1-\lambda),$$

where $\lambda = \frac{\sigma^2}{\sigma^2+\sigma_\varepsilon^2} \in [0,1]$

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Conditional on $\pi = \Pr(\theta = I|\mathcal{I}_U)$, investor U's beliefs are:

$$\mathbb{E}_U[d] = \pi\lambda S_\theta + (1-\pi)0$$

$$\text{var}_U[d] = \underbrace{\pi\sigma^2(1-\lambda) + (1-\pi)\sigma^2}_{\text{expectation of cond. variance}} + \underbrace{\pi(1-\pi)(\lambda S_\theta)^2}_{\text{variance of cond. expectation}}$$

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Note: When U is uncertain about θ, the variance increases with S_θ^2

Benchmark Model: Equilibrium

- Static: Uncertainty about θ, i.e., $\pi \in (0, 1)$, but no learning
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- Optimal θ demand is monotone in S_θ:

$$x_\theta = \frac{\mathbb{E}_\theta[D] - RP}{\alpha \text{var}_\theta[D]} = \frac{\mu + \lambda S_\theta - RP}{\alpha \sigma^2 (1-\lambda)}$$

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- Optimal U demand depends on conditional beliefs:

$$x_U = \frac{\mathbb{E}_U[D] - RP}{\alpha \text{var}_U[D]} = \frac{1}{\alpha}\frac{\mu + \pi\lambda S_\theta - RP}{\pi(1-\lambda)\sigma^2 + (1-\pi)\sigma^2 + \pi(1-\pi)(\lambda S_\theta)^2}$$

- Solve for P using market clearing: $x_U + x_\theta = Z$

Equilibrium price is non-linear in S_θ

$$P = \frac{1}{R}\left(\underbrace{\kappa\mathbb{E}_\theta[D] + (1-\kappa)\mathbb{E}_U[D]}_{\text{expectations}} - \underbrace{\alpha\kappa\sigma^2(1-\lambda)Z}_{\text{risk-premium}}\right)$$

where

$$\kappa = \frac{\text{var}_U\left[d|S_\theta\right]}{\text{var}_U\left[d|S_\theta\right] + \text{var}_\theta\left[d|S_\theta\right]}$$

is increasing in S_θ^2 for $\pi \in (0,1)$, and hump-shaped in π.

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Both components are non-linear in S_θ

- Uncertainty about $\theta \Rightarrow \text{var}_U[d|S_\theta]$ depends on S_θ
- Price is *linear* in S_θ for standard RE / DO models (κ constant)

Asymmetric price reactions to good vs. bad news

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- Good news (positive S_θ) **increases** expectations about fundamentals, but also **increases** U's uncertainty about fundamentals
  $\Rightarrow$ offsetting effects

- Bad news (negative S_θ) **decreases** expectations about fundamentals, and also **increases** U's uncertainty about fundamentals
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Empirical evidence for asymmetric price reactions

- Aggregate level: Campbell and Hentschel (1992)
- Firm level: Skinner (1994), Skinner and Sloan (2002)

Dynamic Model: Setup

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$$x_{i,t} = \frac{\mathbb{E}_{i,t}\left[P_{t+1} + D_{t+1}\right] - RP_t}{\alpha \mathrm{var}_{i,t}\left[P_{t+1} + D_{t+1}\right]}$$

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- Dividends are persistent

$$D_{t+1} = \rho D_t + (1-\rho)\mu + d_{t+1}$$

where $d_{t+1} \sim \mathcal{N}(0, \sigma^2)$ and $\rho < 1$

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- We assume θ_t follows a symmetric Markov switching process with

$$\Pr(\theta_{t+1} = i | \theta_t = i) \equiv q$$

(Also look at i.i.d. case in paper)

Dynamic Model: Learning about θ

Since θ investors are symmetric, U cannot update π_t using $x_{\theta,t}$ and P_t

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But, U can update π_t by comparing realized dividends to $S_{\theta,t}$, i.e.,

$$\pi_{t+1} = \frac{\pi_t \Pr\left(S_{\theta,t} | \theta = I,\ d_{t+1}\right)}{\pi_t \Pr\left(S_{\theta,t} | \theta = I,\ d_{t+1}\right) + (1 - \pi_t) \Pr\left(S_{\theta,t} | \theta = N,\ d_{t+1}\right)}$$

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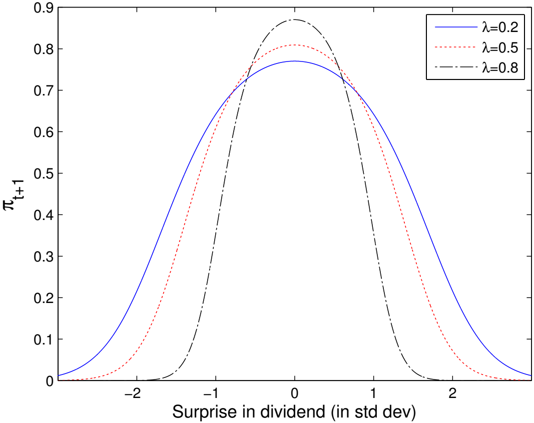
Intuitively:

- When the dividend is in line with the signal $\rightarrow$ increase π_t
- When the dividend is a surprise given the signal $\rightarrow$ decrease π_t

Figure: Updated beliefs after observing P_t, d_{t+1} starting from $\pi_t = 0.75$.



When θ is serially correlated, π_t is stochastic and persistent

Dynamic Model: An Equilibrium Characterization

Proposition: In any signal-revealing equilibrium, the price is

$$P_t = \frac{1}{R}\left(\underbrace{\bar{\mathbb{E}}_t\left[P_{t+1} + D_{t+1}\right]}_{\text{expectations}} - \underbrace{\alpha\kappa_t \mathsf{var}_{\theta,t}\left[P_{t+1} + D_{t+1}\right] Z}_{\text{risk-premium}}\right),$$

where

$$\bar{\mathbb{E}}_t\left[\cdot\right] = \kappa_t \mathbb{E}_{\theta,t}\left[\cdot\right] + (1-\kappa_t)\mathbb{E}_{U,t}\left[\cdot\right]$$

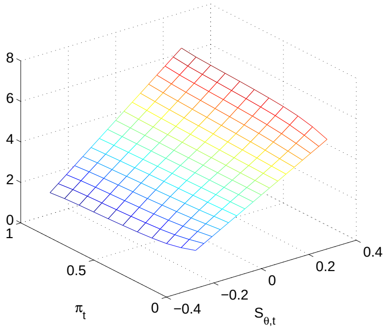
is the *weighted average expectation* of future payoffs, and

$$\kappa_t = \frac{\mathsf{var}_{U,t}[P_{t+1}+D_{t+1}]}{\mathsf{var}_{U,t}[P_{t+1}+D_{t+1}]+\mathsf{var}_{\theta,t}[P_{t+1}+D_{t+1}]} \in (0,1)$$

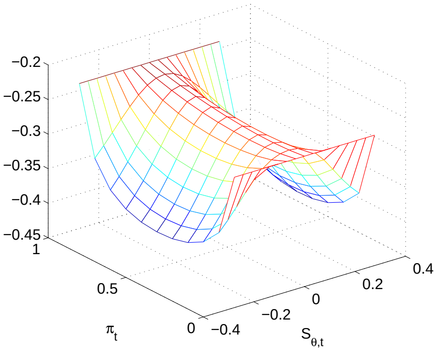
measures the relative precision of the U and θ investors.

Price components in the dynamic model

Similar comparative statics in the dynamic equilibrium



(a) Expectations component



(b) Risk premium component

Implications?

Predictability in expected returns and volatility

Prediction 2: *Learning about other traders leads to stochastic but predictable expected returns and volatility*

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Intuition: Prices and, therefore, return moments depend on π_t

- Updates to π_t depend on $S_{\theta,t}$ and d_t $\Rightarrow$ stochastic moments
- Persistence in π_t $\Rightarrow$ predictability of return moments

Volatility Clustering

Prediction 3: *If* π_t *is sufficiently large, a return surprise (in either direction) predicts higher volatility and expected returns in the future*

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Intuition: π_{t+1} is decreasing in surprises in d_{t+1}

- Recall that, conditional on $\theta = I$, $\mathbb{E}\left[S_{\theta,t} | d_{t+1}\right] = d_{t+1}$
- A large surprise in d_{t+1}, relative to $S_{\theta,t}$, decreases the likelihood that $\theta = I$ i.e., decreases π
- From high π, this implies U faces more uncertainty about θ
- Higher uncertainty $\Rightarrow$ higher expected return and higher volatility

Disagreement and Returns

Prediction 4: *Relation between disagreement and expected returns is non-monotonic and varies over time (with π_t).*

Unlike RE / DO models, disagreement depends on π_t:

$$\mathbb{E}\left(\left|\mathbb{E}_{U,t}\left[D_{t+1}\right] - \mathbb{E}_{\theta,t}\left[D_{t+1}\right]\right)\right. \propto (1-\pi_t)\lambda\sigma$$

When π_t drives disagreement:

- High disagreement (low π_t): Returns **decrease** with disagreement
  Disagreement $\uparrow \quad \Rightarrow \quad \pi_t \downarrow \quad \Rightarrow$ **lower** uncertainty about others
- Low disagreement (high π_t): Returns **increase** with disagreement
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Consistent with Banerjee (2011): high $\pi_t \approx$ RE and low $\pi_t \approx$ DO

Linear return-disagreement relation may be mis-specified!

Helps reconcile the mixed empirical evidence on the return-disagreement relation:

- Negative relation: Diether Malloy Scherbina (2002), Johnson (2004)
- Positive relation: Qu Starks Yan (2004), Banerjee (2011)

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Existing specifications are univariate, linear, and constant over time

Need to control for π_t (e.g., *PIN* (?), institutional ownership)

Parametrization

Show robustness of results

- Theory is in terms of dollar returns i.e., $Q_{t+1} = P_{t+1} + D_{t+1} - RP_t$
- Parametrize the model to generate moments of *rates of return* i.e., $r_{t+1} = Q_{t+1}/P_t$

Get a sense of economic magnitude

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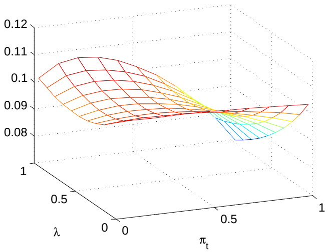
Get a sense of economic magnitude

Set parameters such that for $\pi = 1$ and $\lambda = 0.75$, we have:

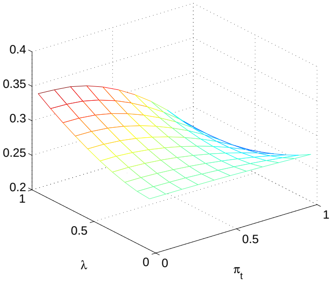
$$\mathbb{E}\left[r_{t+1} - r_f\right] = 7.5\% \text{ and } \sigma\left(r_{t+1}\right) = 22\%$$

- Dividend process: $\mu = 4\%$, $\sigma = 6\%$, $\rho = 0.95$,
- Other parameters: $r_f = 3\%$, $Z = 1$, $\alpha = 1$ and $q = 0.75$.

Return Moments



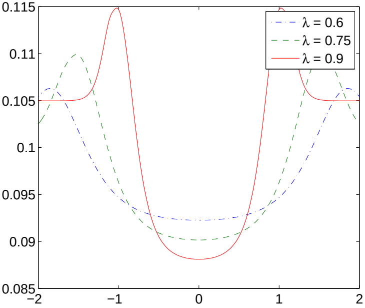
(a) Expected Excess Rate of Return



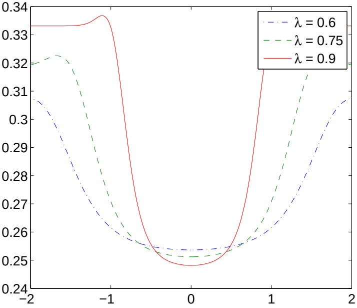
(b) Volatility of Rate of Return

- For $\pi = 1$, change in λ from 0.25 to 0.75
  $\Rightarrow$ exp returns: 9.2% to 7.5%; volatility: 25% to 22%
- For $\lambda = 0.75$, change in π from 1 to 0.5
  $\Rightarrow$ exp returns: 7.5% to 10%; volatility: 22% to 30%

Volatility Clustering



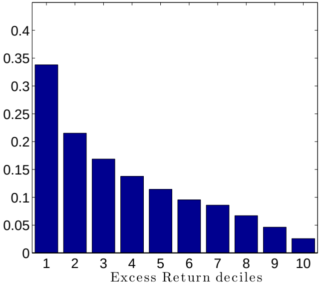
(a) Future Excess Returns



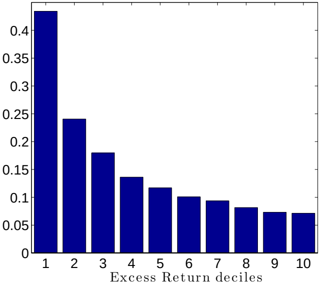
(b) Future Volatility

- For $\pi = 1$ or $\pi = 0$, there is no response
- For $\pi = 0.95$, $\lambda = 0.75$,
  - one std. dev. surprise $\Rightarrow$ exp ret: 9% to 9.6%, vol: 25% to 27%
  - two std. dev. surprise $\Rightarrow$ exp ret: 9% to 10.2%, vol: 25% to 32%

Asymmetric Price Reaction $\Longrightarrow$ Leverage effect



(c) Excess return ($R_{e,t+1}$)



(d) Squared excess return ($R^2_{e,t+1}$)

Returns exhibit reversals: Signals are i.i.d. and short-lived

Asymmetric price reaction: Bigger reversals, higher volatility after negative returns

Robustness: Rational expectations with aggregate noise

Suppose investors have common prior beliefs, but aggregate supply is noisy

- The $\theta = N$ investor knows he does not have information

$$x_\theta = \frac{\mathbb{E}_\theta\left[d\right] - RP}{\alpha \text{var}_\theta\left[d\right]} = \begin{cases} \frac{\lambda S_\theta - RP}{\alpha\sigma(1-\lambda)} & \text{if } \theta = I \\ \frac{0 - RP}{\alpha\sigma} & \text{if } \theta = N \end{cases}$$

- Market clearing condition is given by

$$x_\theta + x_U = Z + z, \quad z \sim N(0, \sigma_z^2)$$

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Suppose investors have common prior beliefs, but aggregate supply is noisy

- The $\theta = N$ investor knows he does not have information

$$x_\theta = \frac{\mathbb{E}_\theta[d] - RP}{\alpha \text{var}_\theta[d]} = \begin{cases} \frac{\lambda S_\theta - RP}{\alpha\sigma(1-\lambda)} & \text{if } \theta = I \\ \frac{0-RP}{\alpha\sigma} & \text{if } \theta = N \end{cases}$$

- Market clearing condition is given by

$$x_\theta + x_U = Z + z, \quad z \sim N(0, \sigma_z^2)$$

- U conditions on P and residual supply $Z + z - x_\theta$ to construct:

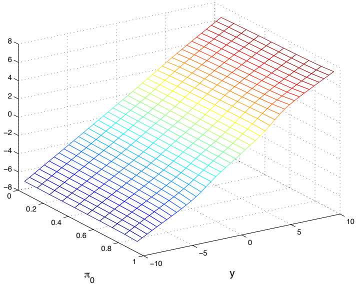
$$y \equiv \alpha\sigma^2(1-\lambda)(x_\theta - z) + P$$

  - Since x_θ is not symmetric, y is informative about θ
  - Conditional on $\theta = I$, y is informative about dividends

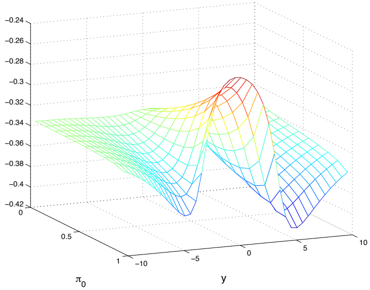
Price decomposition in the Noise Trader Version

Show existence of an equilibrium with price:

$$P = \frac{1}{R}\left(\underbrace{\left(\kappa + (1-\kappa)\pi\lambda_y\right)y}_{\text{expectations}} - \underbrace{\kappa\alpha\sigma^2(1-\lambda)Z}_{\text{risk-premium}}\right)$$



(a) Expectations Component



(b) Risk-premium Component

Summary

Key feature: Uncertainty and learning about whether others are informed

This uncertainty has rich implications for return dynamics

- Non-linear price that reacts asymmetrically to good news vs. bad news
- Stochastic, persistent return moments, even with i.i.d. shocks
- Volatility clustering and the "leverage" effect
- Disagreement-return relation is non-monotonic and time-varying

Model is stylized for tractability and to highlight intuition

- Intuition should be robust to alternative forms of uncertainty
  e.g., about proportion of informed trading
- Future work: Extension to study dynamic information acquisition