(Reverse) Price Discriminaton with Information Design

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Motivation

Three typical ways to price discriminate:

1. Charge price based on customer specific information
2. Offer menu of prices and quantities (or bundles, qualities, etc.) that induce customers to self select
3. Charge different prices to different market segments
   - e.g., discounts for students, children, seniors

In this paper, we propose another channel to facilitate price discrimination: Information Design

The Setting

Monopolist seller facing privately informed buyer

- Buyer's value depends on two components
  1. Private type (θ)
  2. Product quality (ω)
- Seller offers a menu of both prices and experiments
  - Experiments reveal information about ω
  - e.g., free trial, test drive, informative advertisement
- Buyer chooses from the menu, observes the result of the experiment, and then decides whether to buy the good

Example: Menu of Prices and Experiments

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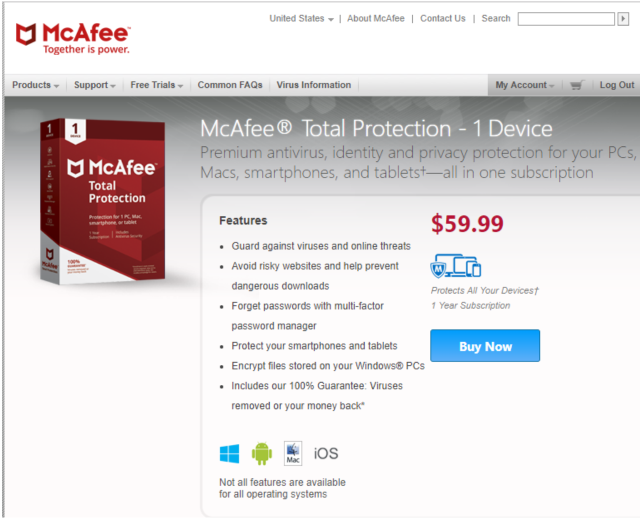


Research Questions

When the seller has control of both price and information,

- What is the optimal sales contract?
- How do price discrimination and information design interact?
- What are the welfare implications when firms can do both?

Preview of Results

The optimal mechanism features:

- Both price discrimination and information discrimination
- Reverse price discrimination: types with higher private values are charged lower prices
- Information is disclosed through stochastic recommendations
  - Each type learns whether ω is above some threshold
  - The threshold is decreasing in θ
  - Thus higher types buy more frequently
- Mechanism remains optimal within a general class of sequential screening mechanisms satisfying ex post IR

Suggestive Evidence

1. Firms charge higher prices from customers after they do a free trial: Gallaugher & Wang (1999), Cheng & Liu (2012)
    - Or, offer discounts to "buy it now"
    - McAfee's menu: $\{(\$35, \text{no trial}), (\$60, \text{30-day trial})\}$
2. Customers who do free trials have lower retention rate: Datta et al. (2015)

Related Literature

- Persuasion: Rayo & Segal (2010), Kamenica & Gentzkow (2011), Kolotilin et al. (2017), etc.
- Sequential screening: County & Li (2000), Krähmer & Strausz (2015), Bergemann, Castro & Weintraub (2017)
- Mechanism design with information disclosure: Eso & Szentes (2007), Li & Shi (2017), Bergemann, Bonatti & Smolin (2018), Krähmer (2018), Smolin (2019)
- Disclosure in Auctions: Milgrom & Weber (1982), Ottaviani & Prat (2001)

Players and Payoffs

- A seller (she) and a buyer (he)
- A single-unit object for sale
- Buyer's willingness to pay $u(\theta, \omega)$
  - $\theta \in \Theta$ is the buyer's personal taste (type)
  - $\omega \in \Omega$ is the product quality (state)
- Given θ, ω, price p, and purchase decision $a \in \{0, 1\}$,

$$u^S = a(p - c),$$
$$u^B = a(u(\theta, \omega) - p),$$

where c is the production cost.

Informational Environment

- The random variables θ and ω are independent
  - $\omega \sim F$ and $\theta \sim G$
- θ is the buyer's private information
- Neither party knows ω, but the seller can design experiments to reveal information about it
  - An experiment is a pair (S, σ)
  - S is a signal space
  - $\sigma : \Omega \to \Delta S$ is a collection of distributions

Examples of Experiments

- Full revelation: $S \equiv \Omega$, $\sigma(\omega) = \omega$
- No revelation: $s = \begin{cases} h, & \text{w.p. } 0.5 \\ t, & \text{w.p. } 0.5 \end{cases}$
- Quality standard: $s = \begin{cases} \text{Success}, & \text{if } \omega \geq \omega^* \\ \text{Failure}, & \text{if } \omega < \omega^* \end{cases}$
- $\cdots$

Timing

- The seller offers and commits to a direct mechanism, which is a menu of price/experiment pairs

$$\{p(\theta), (S_\theta, \sigma_\theta)\}_{\theta \in \Theta}$$

- The buyer privately observes θ, and makes a report $\hat{\theta}$
- A signal $s \in S_{\hat{\theta}}$ is drawn from $\sigma_{\hat{\theta}}$
- The buyer decides whether or not to purchase at price $p(\hat{\theta})$

The Seller's Problem

The buyer will report truthfully iff for all $\theta, \hat{\theta} \in \Theta$

$$\begin{aligned} & \mathrm{E}_{s\sim\sigma_\theta}\left[\max\left\{\mathrm{E}_\omega[u(\theta,\omega)|s,\theta] - p(\theta), 0\right\}\right] \\ \geq\ & \mathrm{E}_{s\sim\sigma_{\hat{\theta}}}\left[\max\left\{\mathrm{E}_\omega[u(\theta,\omega)|s,\hat{\theta}] - p(\hat{\theta}), 0\right\}\right] \end{aligned} \tag{IC}$$

The seller's problem is

$$\max_{(p(\cdot),(S_\cdot,\sigma_\cdot))} \mathrm{E}_\theta\left[(p(\theta)-c)\Pr(\text{type } \theta \text{ buys})\right]$$

$$\text{s.t. (IC)}$$

Plan of attack

- Sufficiency of recommendation mechanisms
- Intuition from a binary model
- Continuous model
  - Characterize IC for recommendation mechanisms
  - Solve relaxed program
  - Verify solution to relaxed program satifies IC
- Welfare implications
- Generalizations

Recommendation Mechanisms

Definition

A **recommendation mechanism** is a direct mechanism that satisfies $S_\theta = \{0, 1\}$ for all $\theta \in \Theta$ and

$$\mathrm{E}[u(\theta, \omega)|s = 1, \theta] - p(\theta) \geq 0$$

$$\mathrm{E}[u(\theta, \omega)|s = 0, \theta] - p(\theta) \leq 0$$

- Information about ω revealed via obedient recommendations.

Lemma

For any IC direct mechanism, there exists an IC recommendation mechanism that generates the same profit to the seller.

Proof

- Take any IC mechanism $\{p(\theta), (S_\theta, \sigma_\theta)\}_{\theta \in \Theta}$
- We will construct an IC recommendation mechanism that generates the same expected profit.
- Continue using $p(\theta)$. For each θ, define

$$\tilde{s}_\theta(s) = \begin{cases} 1, & \text{if } \mathrm{E}_\omega[\theta + \omega - p(\theta) | s] \geq 0 \\ 0, & \text{if } \mathrm{E}_\omega[\theta + \omega - p(\theta) | s] < 0 \end{cases}$$

- The new mechanism is obedient and generates same profit
- It is also IC, because the original mechanism is IC and deviation is less profitable
    - New experiment is Blackwell garbling of original one

Binary Model

Consider the following special case:

- $\Omega = \{0, 1\}$, where $\mu = \Pr(\omega = 1)$
- $\Theta = \{\theta_L, \theta_H\}$
- $u(\theta, \omega) = \theta + \omega$

Benchmarks

1. No Information Design $\implies$ No Price Discrimination
   - Suppose seller can only offer menu of prices and quantities,
   - Then, the optimal mechanism can be implemented with a single fixed price (Myerson 1981, Bulow & Roberts 1989)
2. No Price Discrimination $\implies$ No Information Discrimination
   - Suppose the price is uniform across types (i.e., $p_H = p_L$)
   - Then, the optimal mechanism can be implemented with public disclosure (Kolotolin et. al, 2017)

Price Discrimination with Information Design?

For simplicity and to avoid trivial cases

Assumption

- $0 = c = \theta_L,\ \theta_H < \mu,\ \textit{and}\ \Pr(\theta_H) \leq \frac{\mu - \theta_H}{\mu}$

Without price discrimination, the optimal mechanism involves

- $p^* = \theta_L + \mathrm{E}[\omega] = \mu$
- No information disclosure
    - Both types buy w.p.1
    - Seller extracts all surplus from low type
    - High type surplus is θ_H

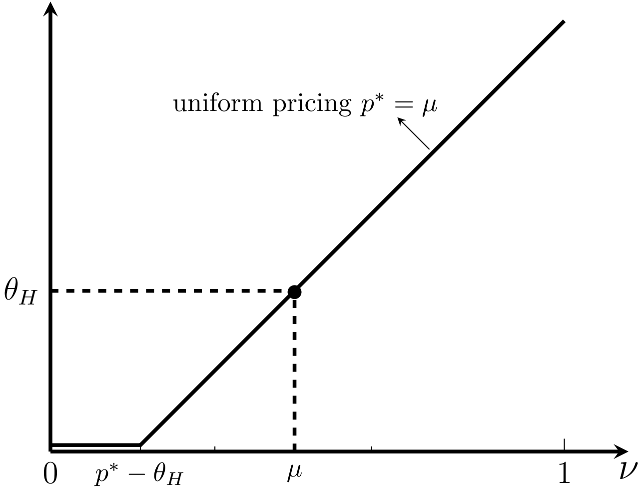
Let's see how seller can extract more surplus once both instruments can be utilized...

A Better Mechanism

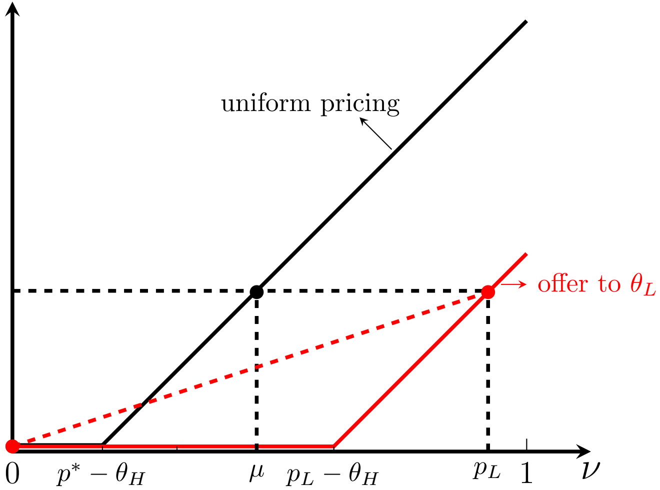
Consider a perturbation in which the seller offers the low type

- A higher price $p_L > p^*$
- The "Bayesian Persuasion" disclosure policy, where the posterior after observing s is
  - $\Pr(\omega = 1 | s = 1) = p_L$
  - $\Pr(\omega = 1 | s = 0) = 0$
- This offer still extracts all of the surplus from the low type
  - And relaxes the IC constraint of the high type
  - Which allows the seller to charge θ_H a higher price

High-type Indirect Utility



High-type Indirect Utility



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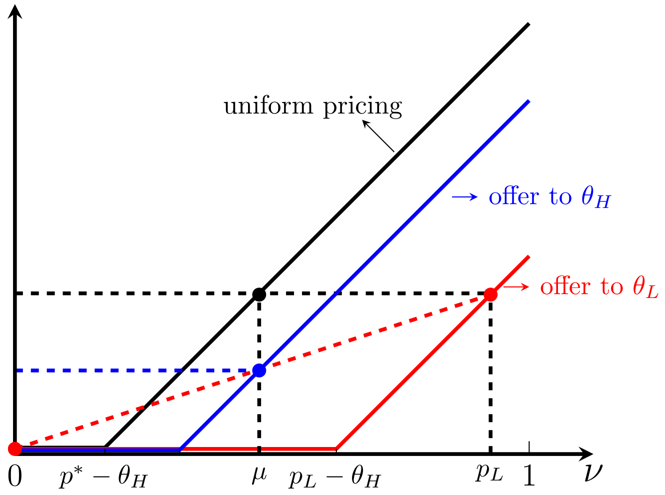


Optimal Mechanism

The optimal mechanism employs the perturbation to its extreme by increasing p_L to the highest possible price.

Proposition

The optimal mechanism in the binary model involves

- $p_L^* = 1$ *and* ω *is fully revealed to* θ_L
- $p_H^* < 1$ *and no information about* ω *is revealed to* θ_H

Three Generalizable Features

1. The seller employs **both** price discrimination and information discrimination,
   - Information design activates a role for price discrimination
2. Higher types are offered lower prices
3. When recommended to buy, lower types have higher expectations about quality

Continuous Types and States

Suppose now that:

- $\Omega = [\underline{\omega}, \bar{\omega}]$, $\omega \sim F$, with pdf f
- $\Theta = [\underline{\theta}, \bar{\theta}]$, $\theta \sim G$, with pdf g
- Retain additive valuation: $u(\theta, \omega) = \theta + \omega$
  - Assume $\underline{\theta} + \underline{\omega} \leq c < \bar{\theta} + \bar{\omega}$

Assumption (Monotone hazard rate)

The hazard rate, $\frac{g(\theta)}{1-G(\theta)}$, is strictly increasing on Θ.

Recommendation Mechanisms

A recommendation mechanism can be written as

$$\{p(\theta), q(\omega, \theta)\}_{\theta \in \Theta}$$

where $q(\omega, \theta)$ is the probability of $s = 1$ given (ω, θ)

- There are two types of IC constraints for recommendation mechanisms
    1. Truthful reporting: willing to report type truthfully
    2. Obedience: conditional on truthful reporting, willing to follow recommendation
- Need to handle "double deviations"
    - First lying, then disobeying

IC for Recommendation Mechanisms

If a type θ buyer reports truthfully and obeys:

$$V(\theta) = \int_\Omega (\theta + \omega - p(\theta))q(\omega, \theta)dF(\omega)$$

Relevant deviations

(i) Reporting $\hat{\theta}$ and then obeying: $U(\theta, \hat{\theta})$

(ii) Reporting $\hat{\theta}$ and always buying: $\theta + \mu - p(\hat{\theta})$

(iii) Reporting $\hat{\theta}$ and never buying: 0

If (ii) and (iii) are suboptimal then so is (iv) always disobeying.

IC for Recommendation Mechanisms

Therefore, incentive compatibility can be written as

$$V(\theta) \geq \max_{\hat{\theta} \in \Theta} \left\{ 0, \theta + \mu - p(\hat{\theta}), U(\theta, \hat{\theta}) \right\}, \text{ for all } \theta, \hat{\theta} \in \Theta \qquad \text{(IC)}$$

- The seller's program is

$$\max_{\{p(\theta), q(\omega,\theta)\}} \mathrm{E}_{\theta} \left[(p(\theta) - c) \int_{\underline{\omega}}^{\bar{\omega}} q(\omega, \theta) dF(\omega) \right]$$
$$\text{s.t. (IC)}$$

Solution Technique

- We first solve a relaxed program ignoring double deviations

$$\max_{\{p(\theta),q(\omega,\theta)\}} \; \mathrm{E}_\theta\left[(p(\theta)-c)\int_\Omega q(\omega,\theta)dF(\omega)\right]$$

$$\text{s.t. } V(\theta) \geq U(\theta,\hat{\theta}), \forall\theta,\hat{\theta} \quad \text{(IC-relaxed)}$$

- We then verify the candidate solution satisfies (IC)

Solving the Relaxed Program

To solve the relaxed program, we follow the standard approach. Fix a q:

- Envelope theorem implies the information rent to each type
- Can back out the price function needed to satisfy (IC-relaxed)
- Write the seller's program in terms of q only
- Integrate by parts

Solving the Relaxed Program

The seller's program becomes

$$\max_{\{q(\omega,\theta)\}} \int_\Theta \int_\Omega \left(\omega + \theta - \frac{1-G(\theta)}{g(\theta)} - c\right) q(\omega,\theta) dF(\omega) dG(\theta)$$

$$\text{s.t. } B(\theta) = \int_\Omega q(\omega,\theta) dF(\omega) \text{ is nondecreasing}$$

Pointwise optimization suggests a candidate solution for the relaxed program:

$$q^*(\omega,\theta) = \begin{cases} 1, & \text{if } \omega \geq m(\theta) \\ 0, & \text{if } \omega < m(\theta) \end{cases}$$

where $m(\theta) \equiv -\left(\theta - \frac{1-G(\theta)}{g(\theta)} - c\right)$ is inverse of virtual surplus.

Candidate Solution

Under this candidate, type θ is recommended to buy

- When quality exceeds the threshold $m(\theta)$
- With probabiliity $B^*(\theta) = 1 - F(m(\theta)) = \Pr(\omega \geq m(\theta))$

The pricing function implied by the envelope condition is:

$$p^*(\theta) = \theta + \mathrm{E}(\omega | \omega \geq m(\theta)) - \frac{\int_{\underline{\theta}}^{\theta} B^*(s) ds}{B^*(\theta)}$$

Main Result

Theorem

Suppose the monotone hazard condition holds. Then $\{p^*, q^*\}$ is the (essentially) unique optimal recommendation mechanism.

In this mechanism, higher types

(i) Buy more often (B^* is increasing), and

(ii) Pay a lower price conditional on buying (p^* is decreasing).

The monotone hazard condition plays a dual role: implies solution to the relaxed program satisfies both (i) and (ii).

- Sufficient to rule out double deviations.

Ruling out Double Deviations

Recall the original IC constraint:

$$V(\theta) \geq \max_{\hat{\theta}\in\Theta} \left\{0, \theta + \mu - p^*(\hat{\theta}), U(\theta, \hat{\theta})\right\} \tag{IC}$$

In $\{p^*, q^*\}$:

- In order to get the lowest price, the buyer should report $\bar{\theta}$
- And $\bar{\theta}$ is always recommended to buy
- So "reporting $\bar{\theta}$ and always buying" is equivalent to "reporting $\bar{\theta}$ and obeying"

Intuition for RPD

Two familiar properties of the optimal mechanism

(1) Each type buys less often than is efficient, but higher types buy *more often* (standard in mechanism design)

- Quality threshold $m(\theta)$ is decreasing in θ

(2) Each type buys *more often* than when he can perfectly observe the state and faces the same price (standard in persuasion)

- Quality threshold is below $p^*(\theta) - \theta$

(1) and (2) require a decreasing price to induce truthful reporting

- If price was weakly increasing, buyer should report lower type and get a more preferable (i.e., a higher) quality threshold

Welfare Implications

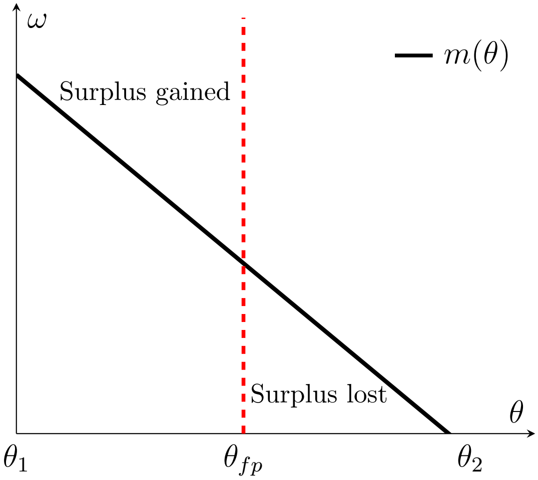
Exercise: Compare the payoffs under the optimal mechanism with information design to those in the canonical optimal mechanism without information design (i.e., a fixed price).

- Clearly, the seller benefits from information design.
- What about the consumer? total surplus?

For this exercise, we shut down any social value of information.

- Efficient outcome is to trade for all (θ, ω)

Welfare Implications



Welfare Implications

Information design facilitates:

- Surplus creation on the extensive margin as the seller serves more types with positive probability.
- Surplus destruction on the intensive margin as customer just above θ_{fp} buy with probability less than one.

Integrating over all types, both total surplus and consumer surplus may increase or decrease

- Examples of both in the paper

Generalizations

- Contractible signals
- Relaxing the monotone hazard condition
  - Alternative sufficient condition: $p(\bar{\theta}) \leq p(\theta)\ \forall \theta$
  - When this fails...???
- More general valuation functions
  - Need bounds on the cross partial
- Endogenous quality (à la Mussa and Rosen, 1979)
  - Higher types recieve higher quality, may face higher prices
- More general screening mechanisms

General Mechanisms

- Consider a more general class of mechanisms in which the seller elicits information of type and signal realization in two stages

$$\{(S_\theta, \sigma_\theta), (X(\theta, s), T(\theta, s))\}_{\theta \in \Theta}$$

- The buyer first reports a type θ to get an experiment $(S_\theta, \sigma_\theta)$
- The buyer then observes the signal realization and reports s
- Given the reported type and signal, the buyer pays $T(\theta, s)$ and gets the object with probability $X(\theta, s)$

General Mechanisms

- Eso and Szentes (2007) and Li and Shi (2017) studied this problem
- In the independent case, the optimal (interim IR) mechanism:
  1. **Fully** discloses the state ω to **all** types
  2. charges an entry fee $c(\theta)$, and a premium $p(\theta)$ on buying
- Note that this mechanism is not **ex post IR**
  - European directive 2011/83/EU mandates withdrawl rights for internet sales

General Mechanisms

Result

The mechanism described in the Theorem is an optimal ex post IR general mechanism.

- In the optimal interim IR mechanism, the seller provides uniform and maximal information to all buyers
- In the optimal ex post IR mechanism, the seller provides differential and partial information across different buyers

Conclusion

- We study the optimal sales contract when the seller can control both price and information about product quality
- The optimal mechanism features
  - reverse price discrimination, and
  - discriminatory information disclosure
- We highlight the complimentarity between price discrimination and information discrimination
- The mechanism remains optimal with contractible signals and within in general class of mechanisms satisfying ex post IR

Contractible Signals

- What if the seller can observe the realization of s?
- A direct mechanism is now $\{p(\theta, s), (S_\theta, \sigma_\theta)\}$

Result

The mechanism described in Theorem 1 remains optimal even if signals are contractible.

Contractible Signals

Proof Sketch

- It is still WLOG to restrict to recommendation mechanisms with contractible signals: $\{p(\theta,1), p(\theta,0), q(\omega,\theta)\}_{\theta\in\Theta}$
- The price $p(\theta,0)$ is never actually paid, but relaxes the IC constraint

$$V(\theta) \geq \max_{\hat{\theta}}\{0, U(\theta,\hat{\theta})\}, \text{ for all } \theta \tag{IC-contractible}$$

- (IC) $\Rightarrow$ (IC-contractible) $\Rightarrow$ (IC-relaxed)