Dynamics, Asymmetric Information and News

Brett Green

Designed for PhD course on Daley and Green (2012)

Overview

The literature studies a plethora of static environments where agents have private information.

- Important to distinguish between mechanism design/contract theory (optimal incentive schemes) and competitive market equilibria. We'll focus on the latter today.
- The seminole examples in economics are Akerlof (1970), Spence (1973) and their finance counterparts Myers and Majluf (1984) and Leland and Pyle (1977).
- A natural question to ask is: How robust are the predictions of our static models to dynamic environments? This is what we will explore today.
- Then we will investigate the impact that gradual information revelation has on trade dynamics.

The Basic Setting

Single seller with asset of type $\theta \in \{L, H\}$

- Asset is H (w.p. π), L (w.p. $1 - \pi$)
- Seller privately knows θ, buyers do not
- Seller has flow value k_θ, buyers derive flow value v_θ.
- Common knowledge of gains from trade: $k_\theta < v_\theta$
- All agents are risk neutral and have common discount rate r

Preliminaries

- We use this setup to analyze both Akerlof and Spence. Then investigate their dynamic counterpart.
- Seller's outside option (i.e., if she never trades):

$$K_\theta = \int_0^\infty e^{-rs} k_\theta ds = \frac{k_\theta}{r}$$

- Next, consider how much a buyer is willing to pay assuming both types sell:

$$E[V_\theta] = \pi V_H + (1-\pi) V_L$$

where $V_\theta = \frac{v_\theta}{r}$.

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Akerlof's Market for Lemons

The extensive form:

- There is a single date at which trade can occur ($t = 0$).
- Multiple (> 2) buyers arrive and make-take-it-or-leave-it offers. Note: Buyers compete ala Bertrand so make zero-profit.
- If seller rejects, her payoff is K_θ.
- If seller accepts an offer of w, her payoff is just w.
- Winner buyer's payoff is $w - V_\theta$. Losing buyers make zero.

What is the equilibrium outcome?

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Two cases:

1. $K_H < E[V_\theta] \implies$ Market for Everything
   - Equilibrium price is $w = E[V_\theta]$
   - Both types trade w.p.1. $\implies$ Outcome is efficient.
2. $K_H > E[V_\theta] \implies$ Market for Lemons
   - Hit won't sell for less than K_H.
   - Any $w \geq K_H$ will lose money on average.
   - Hence $w < K_H$. Zero profit implies $w = V_L$.
   - Hits don't trade. Letdowns sell for V_L w.p.1. $\implies$ Outcome is inefficient.

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Is the prediction from the Market for Lemons ($K_H > E[V_\theta]$) robust to a dynamic setting?

- What happens the next day ($t = 1$)?
  - There will be more buyers
  - Only H is left—price should increase to V_H
  - Letdown's regret their decision to sell at $t = 0$
  - The equilibrium unravels

Note: Similar criticism applies to Myers and Majluf. If only certain types of firms issue equity and invest at $t = 0$, the decision not to issue/invest reveals information about the firms type. Beliefs/price tomorrow will be different.

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Spence's Market Signaling

The extensive form:

- Seller can *commit* to any amount of costly delay: strategy is a mapping $\sigma : \Theta \to \Delta(\mathbb{R}_+)$.
- Buyers observe seller's action ($t \in \mathbb{R}_+$) and update their beliefs from π to $\mu(t)$.
- Buyers simultaneously and make offers at date t. Buyer i's strategy is a mapping $w_i : \mathbb{R}_+ \to \Delta(\mathbb{R})$.
- Seller decides which offer to accept (if any).

Spence's Market Signaling

Payoffs:

- To seller from trading at time t for price w:

$$u_\theta(t, w) = \int_0^t e^{-rs} k_\theta ds + e^{-rt} w = (1 - e^{-rt}) K_\theta + e^{-rt} w$$

- To buyers:

$$\begin{array}{ll} V_\theta - w & \text{if trades with type } \theta \text{ at price } w \\ 0 & \text{if does not trade} \end{array}$$

- Again, seller will be paid his expected value based on Buyers' beliefs (i.e., Bertrand competition drives profits to zero).

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Spence's Market Signaling

Typical parametric restriction in "signaling" environments:

- $K_H < V_L$: No adverse selection problem
- $K_L < K_H$: Single-crossing condition

Under these conditions, the model admits two types of equilibria:

1. Full Pooling: $\sigma_L = \sigma_H = t_p$ for any t_p such that

$$u_L(t_p, E[V_\theta]) \geq V_L \Leftrightarrow t_p \leq \frac{1}{r} \ln\left(\frac{V_L - K_L}{E[V_\theta] - K_L}\right)$$

2. Separating: $\sigma_L = 0$, $\sigma_H = t_H$, where t_H is such that

$$u_L(0, V_L) = V_L \geq u_L(t_H, V_H) \Leftrightarrow t_H \geq \frac{1}{r} \ln\left(\frac{V_L - K_L}{V_H - K_L}\right)$$

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Equilibrium Selection in Signaling Models

- To make sharp predictions, we are left with the (difficult) task of selecting which equilibrium is "most reasonable."
- Standard equilibrium refinements (Intuitive Criterion, D1, Universal Divinity), which are based on refining the set of off-equilibrium path beliefs, all arrive at the same conclusion.

Proposition

The unique equilibrium outcome satisfying standard refinements is the least-cost-separating equilibrium. That is, $\sigma_L = 0$ *and* $\sigma_H = t^*$ *where* t^* *is such that* $u_L(0, V_L) = u_L(t^*, V_H)$.

Intuition: From indifference curves (optional)

Separating Equilibria in Dynamic Settings

Question: Is this prediction robust to a dynamic setting?

- If only H waits to sell, type is revealed by not selling at $t = 0$...
  - Buyers should snatch up any seller that remains for $V_H - \epsilon$
  - L regrets decision to sell at $t = 0$
  - Equilibrium again unravels in a dynamic setting

Note: A similar criticism applies to Leland and Pyle.

- If a firm reveals its type through how much equity it retains at $t = 0$, then it should be able to sell the rest of the equity at fair market value at $t = 1$.
- But then low type firm who sold all their equity at $t = 0$ will regret their decision and the equilibrium unravels.

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Akerlof vs Spence

These static models are quite different:

- Akerlof - seller chooses whether to **trade now or never**
  - Somewhat stark as mentioned previously.
- Spence - seller can **commit** to any amount of costly delay
  - Is this commitment power reasonable? What stops buyers from making offers sooner?

Observation

In a dynamic market (without commitment), these two strategic settings are virtually identical!

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A Dynamic Market with Asymmetric Information

Reformulated version:

- Seller is interested in selling her asset but she is not forced to do so on any particular day
- If she does not sell today, she derives the flow value from the asset
- And can entertain more offers tomorrow

The Model: Daley and Green (2012)

Players:

- Initial owner, A_0
- Mass of potential buyers (the market)

Preferences:

- All agents are risk-neutral
- Buyers have discount rate r
- A_0 discounts at $\bar{r} > r$

The Model

The Asset:

- Single asset of type $\theta \in \{L, H\}$
- Nature chooses θ: $\mathbb{P}(\theta = H) = P_0$
- A_0 knows θ and accrues (stochastic) flow payoff with mean v_θ
- High-value asset pays more: $v_H > v_L$
- Let $V_\theta \equiv \int_0^\infty v_\theta e^{-rt} dt$ and $K_\theta \equiv \int_0^\infty v_\theta e^{-\bar{r}t} dt$
- Assume that $K_H > V_L$ (SLC)

News Arrival

- Brownian motion drives the arrival of *news*. Both type assets start with the same initial score X_0
- Type θ asset has a publicly observable *score* (X_t^θ) which evolves according to:

$$dX_t^\theta = \mu_\theta dt + \sigma dB_t$$

where B is standard B.M. and WLOG, $\mu_H \geq \mu_L$

- The *quality* (or *speed*) of the news is measured by the signal-to-noise ratio: $\phi \equiv \frac{\mu_H - \mu_L}{\sigma}$
- You can think about ϕ as either a quality: how much can be learned in a certain amount of time or as a rate: how fast can something be learned. It is a sufficient statistic for the drift terms and the volatility.
- One interpretation:
  - News=cashflows or information about them: $\mu_\theta = v_\theta$

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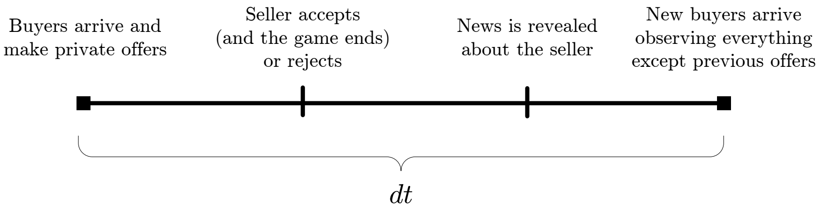
Timing

- Infinite-horizon, continuous-time setting
- At every t:
  - Buyers arrive and make offers.
  - Owner decides which offer to accept (if any).
  - News is revealed about the asset.



Payoffs

- To the initial owner who accepts an offer of w at time t is:

$$\int_0^t e^{-\bar{r}s} v_\theta ds + e^{-\bar{r}t} w = (1 - e^{-\bar{r}t}) K_\theta + e^{-\bar{r}t} w$$

- To a buyer who purchases the asset for w at time t:

$$V_\theta - w$$

Market Beliefs and Owner's Status

Buyers begin with common prior: $\pi = \mathbb{P}_{t=0}(\theta = H)$

- At time t, buyers know:
  (i) The path of news arrival and shocks up to time t
  (ii) That trade has not occurred prior to t

Equilibrium beliefs will conditioned on all of the above.

- Let P denote the equilibrium belief process
- Define $Z = \ln\left(\frac{P}{1-P}\right)$: "beliefs" in z-space

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Strategies and Equilibria

We will construct a stationary equilibrium of the game.

- The **state**, is the market belief z. Any history such that
  - Market beliefs are z: $Z_t(\omega) = z$
- Note that the state evolves endogenously over time— since beliefs must be consistent with strategies.

- Buyers' strategy is summarized by the function w
  - $w(z)$ denotes the (maximal) offer made in state z
- The owner's strategy is a stopping rule τ. Roughly, a mapping from (θ, z, w) into a decision of whether to "stop" (i.e., accept).

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Strategies and Equilibria

Given w and Z, the owner's problem is to choose a stopping rule to maximize her expected payoff given any state.

$$\sup_{\tau} E_t^{\theta}\left[\int_0^{\tau} v_{\theta} e^{-\bar{r}s} ds + e^{-\bar{r}\tau} w(Z_{\tau}) \Big| Z_t\right] \qquad (SP_{\theta})$$

Definition

An equilibrium is a triple (τ, w, Z):

- Given w and Z, the owner's strategy solves SP_{θ}.
- Given τ and Z, w is consistent with buyers playing best responses.
- Market beliefs, Z, are consistent with Bayes rule whenever possible.

Evolution of Beliefs based only on News

A useful element in the equilibrium construction is the belief process that updates only based on news:

- Starting from the initial prior P_0
- Let $\hat{P}$ denote the belief process resulting from Bayesian updating *based only on news*:

$$\hat{P}_t \equiv \frac{\pi f_t^H(X_t)}{\pi f_t^H(X_t) + (1-\pi) f_t^L(X_t)}$$

  - Where f_t^θ is the normal pdf with mean $\mu_\theta t$ and variance $\sigma^2 t$
  - The evolution of $\hat{P}$ is type dependent

Why is this useful? If strategies call for trade with probability zero over any interval of time, then the equilibrium (consistent) beliefs must evolve according to $\hat{P}$ over that interval.

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Using Ito's lemma and the law of motion of dX_t^θ gives:

$$d\hat{Z}_t^H = \frac{\phi^2}{2}dt + \phi dB_t$$

$$d\hat{Z}_t^L = -\frac{\phi^2}{2}dt + \phi dB_t$$

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Equilibrium Beliefs

In equilibrium, the market beliefs evolves based on news as well as:

- The owner's equilibrium strategy and the fact that trade has not yet occurred.

That is,

$$dZ_t = d\hat{Z}_t + dQ_t$$

Where dQ_t is the information contained in the fact that trade occurred or did not occur at time t.

For example, suppose trade does not occur at time t:

- If strategies call for trade with probability zero: $dQ_t = 0$
- If strategies call for a low type to trade with positive probability and a high type to trade with probability zero: $dQ_t > 0$

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Note: We have just linearized Bayesian updating (very convenient).

Finding the Equilibrium

Some preliminaries

Proposition

Let $F_\theta(z)$ denote the seller's value function. Properties that must be true of any equilibrium:

1. *Buyers make zero expected profit*
2. $F_L(z) \geq V_L$ *and* $F_H(z) \geq E[V_\theta|z]$ *for all* z
3. $F_L(z) \leq E[V_\theta|z]$ *and* $F_H(z) \leq V_H$ *for all* z
4. *The only prices at which trades occurs are* V_L *and* $E[V_\theta|z]$
5. *If* $E[V_\theta|z]$ *is offered, it is accepted with probability 1*

Equilibrium for $\phi > 0$

Main Result:
There exists an equilibrium of the game which is characterized by a pair of beliefs (α, β) and the function s.t.

- If $z \geq \beta$: $w = B(z)$ and both type sellers accept with probability one.
- If $z \leq \alpha$: $w = V_L$, a high type seller rejects with probability one and a low type seller accepts with probability $\rho_L = 1 - e^{z-\alpha}$.
- If $z \in (\alpha, \beta)$: no trade occurs. Buyers make offers that are rejected by both type sellers with probability one.

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Sample Path of Equilibrium Play

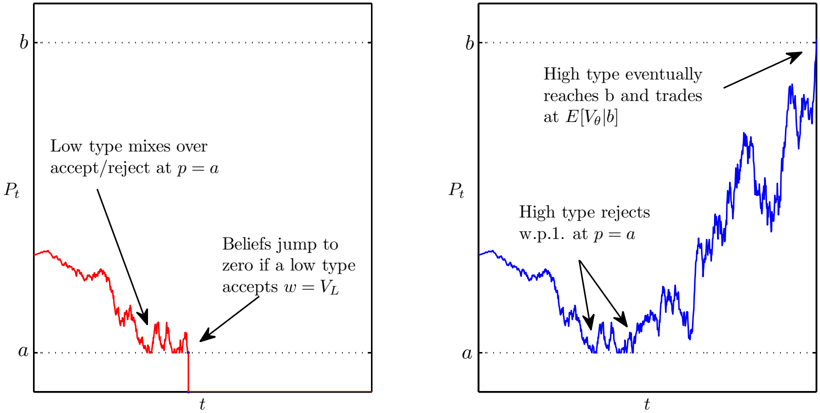

Figure : A low type may eventually sell for V_L at $p = a$ (left), a high type never does (right). Notice that the low type accepts in such a way that the equilibrium beliefs reflect of the lower boundary. That is Z has a lower reflecting barrier at α.

Proof By Construction

Take w as given and assume that Z evolves as specified for some unknown α.

- Let $F_\theta(z)$ denote the *seller's* value for the asset. The Bellman equation for the seller's problem is:

$$F_\theta(z) = \max\left\{ \overbrace{w(z)}^{\text{payoff from accepting}}, \underbrace{v_\theta dt + \mathbb{E}^\theta\left[e^{-\bar{r}dt}F_\theta(z+dZ)\right]}_{\text{payoff from rejecting}} \right\}$$

- In the no-trade region:

$$F_\theta(z) = v_\theta dt + \mathbb{E}^\theta\left[e^{-\bar{r}dt}F_\theta(z+dZ)\right]$$

- And Z evolves only based on news, implying that $\forall z \in (\alpha, \beta)$, F_θ satisfies a second-order ODE. The two ODEs have simple closed-form solutions e.g.,

$$F_L(z) = c_1 e^{u_1 z} + c_2 e^{u_2 z} + K_L$$

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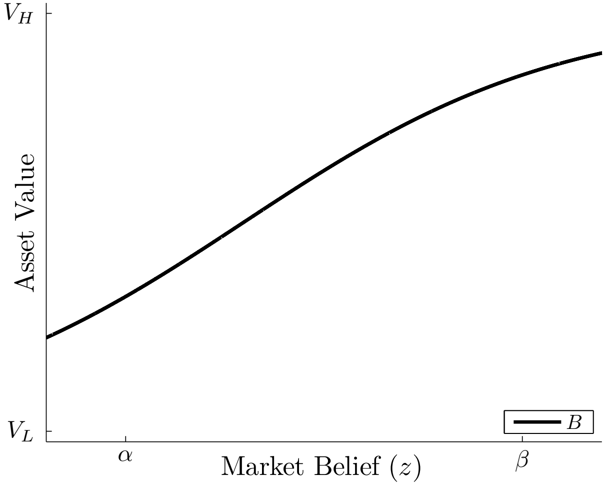

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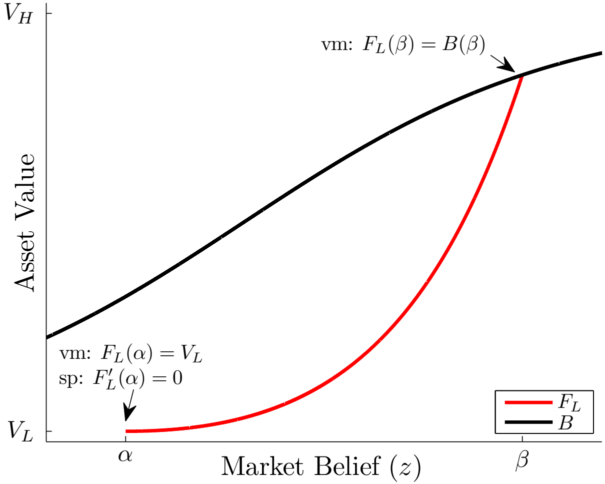

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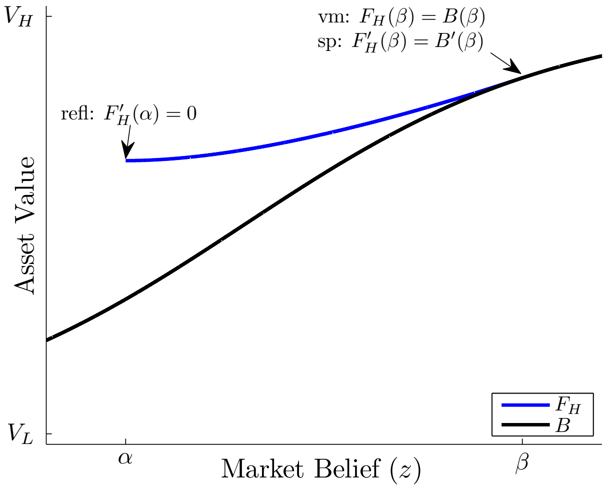

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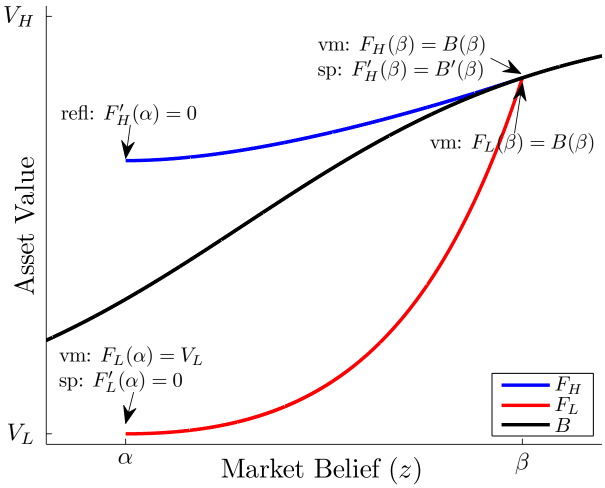

Solution to the Seller's Problem

Lemma

There exists a unique (α^*, β^*) that simultaneously solves the high and low-type sellers' problem optimal stopping problem.

Proof:

- For any α (i.e., lower barrier on Z), there exists a unique β such that the stopping rule $\tau_H = \inf\{t : Z_t \geq \beta\}$ solves SP_H. Call this mapping $\beta_H(\alpha)$.
- Similarly, for each β, there exists a unique α such that both τ_H and $\tau_L = \inf\{t : Z_t \notin (\alpha, \beta)\}$ solve SP_L (i.e., the low type is indifferent between playing τ_H and τ_L). Call this mapping $\beta_L(\alpha)$.
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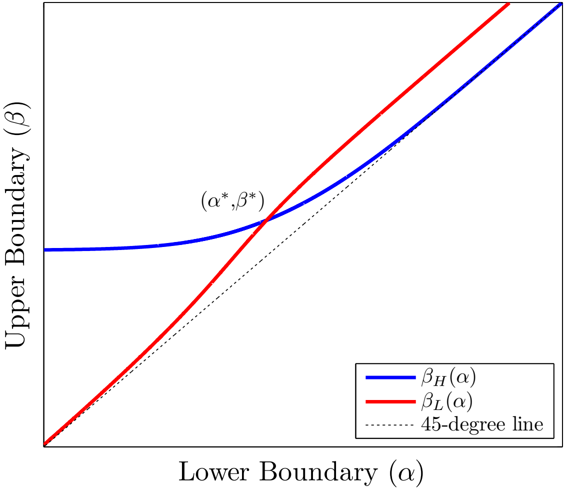
Intersection of β_L and β_H

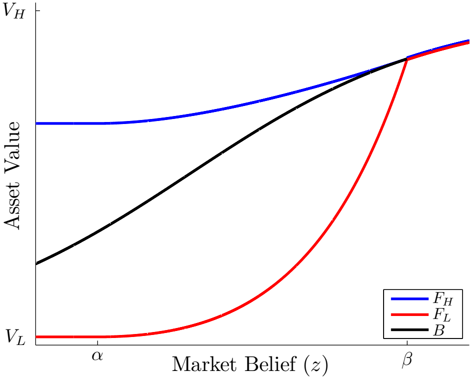
Each curve represents a solution to a class of stopping problems:



Completing the Seller Value Functions

Outside the no-trade region, the seller's value function is as follows:

$$F_L(z) = F_L(\alpha^+), \qquad F_H(z) = F_H(\alpha^+) \qquad \forall z \leq \alpha$$
$$F_L(z) = B(z), \qquad F_H(z) = B(z) \qquad \forall z \geq \beta$$



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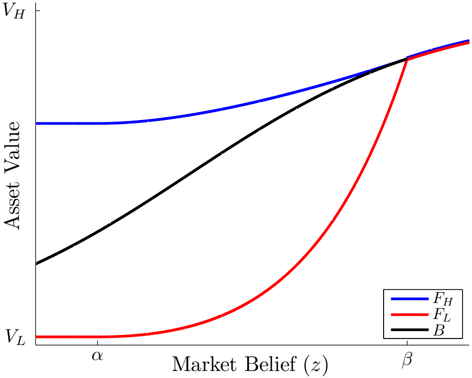
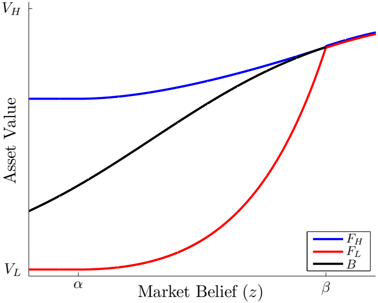


Why is there no trade for $z \in (\alpha, \beta)$?



Intuition for the no-trade region?

- H can always get B if she wants it. This endows H with an option. For $z \in (\alpha, \beta)$, H does better by not exercising the option.
- L can always get V_L if he wants it. But for $z \in (\alpha, \beta)$, L does better to mimic H.

Equilibrium Beliefs at $z = \alpha$

- $F_L(\alpha) = V_L$, $F_L'(\alpha) = 0 \implies$ low-type seller is just indifferent
- Seller cannot accept with an atom at $z = \alpha$
- On the other hand, Z cannot drift below α

Proposition

The low-type sells at a flow rate s/σ proportional to dX_t at $p = a$ such that:

- *Z reflects off $z = \alpha$ if trade does NOT occur*
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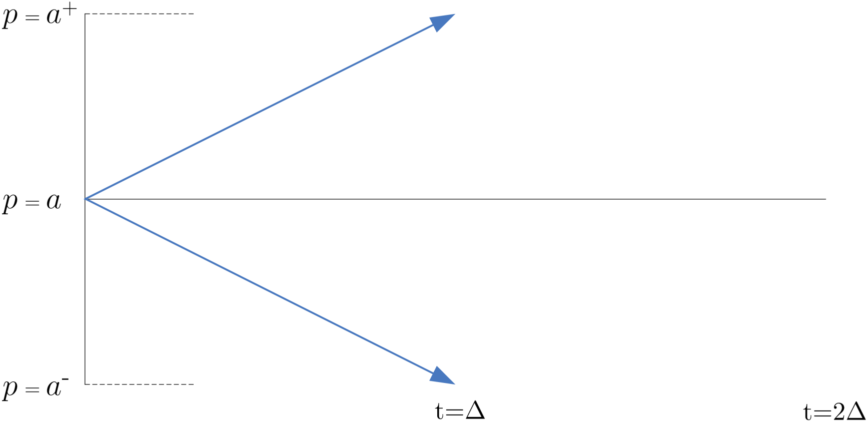
Reflection in Discrete Time



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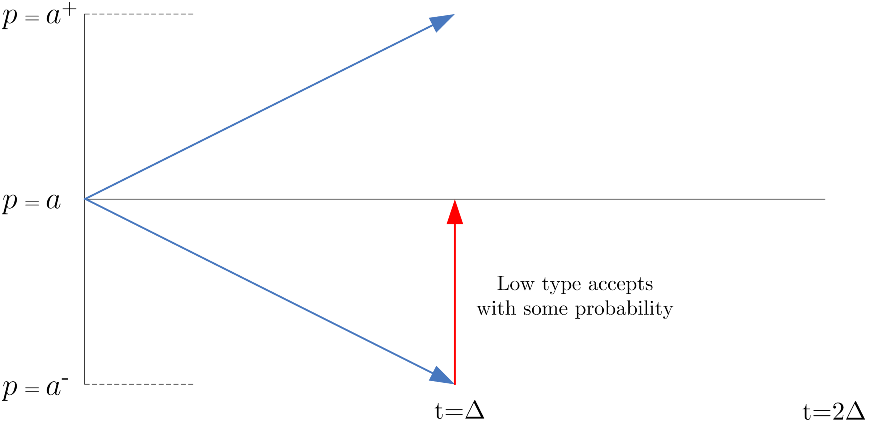

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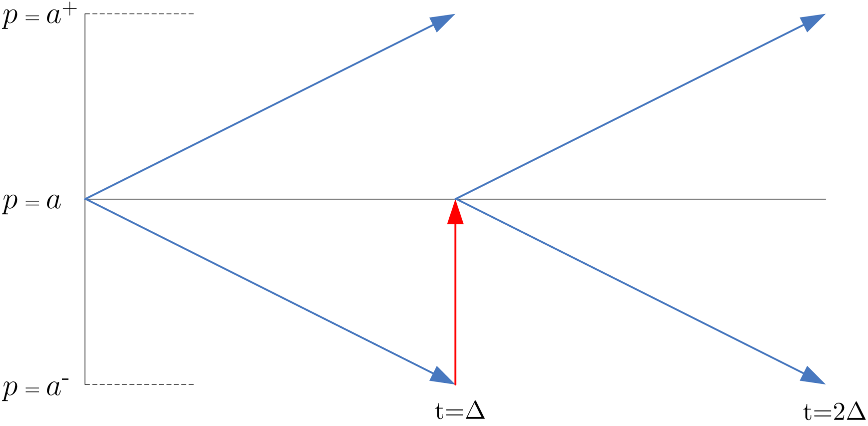

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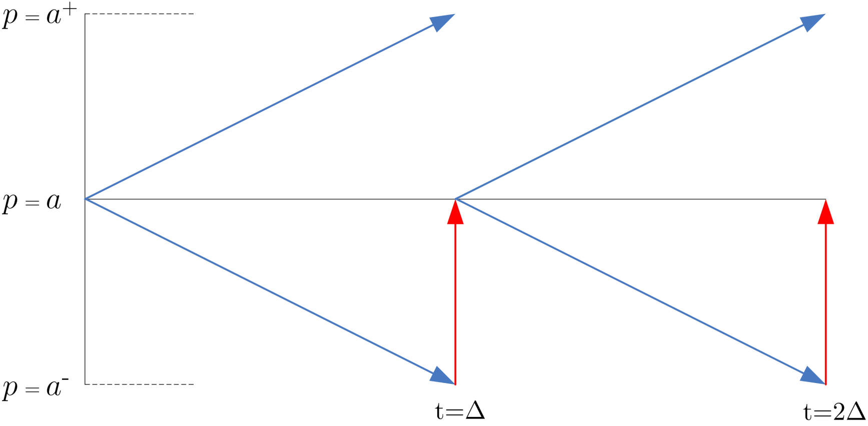
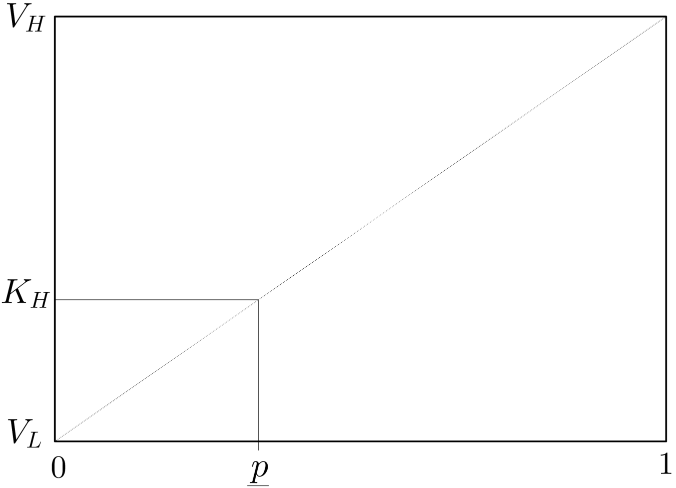


Uniqueness Argument



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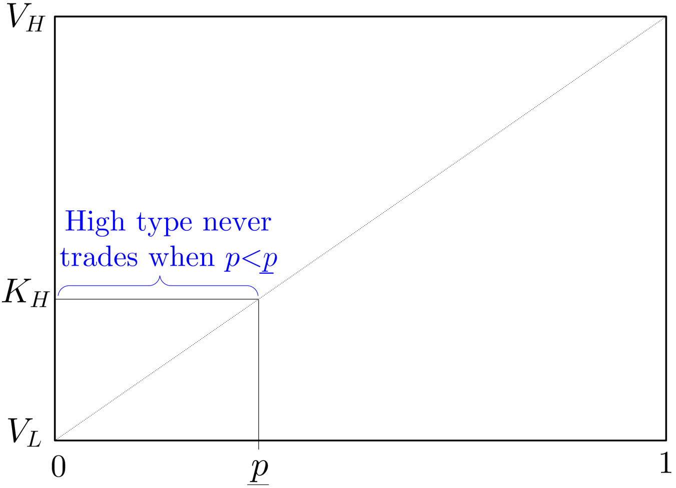

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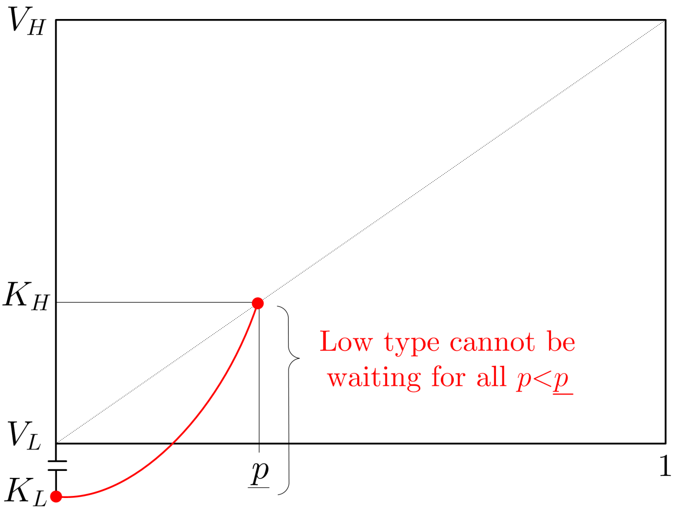

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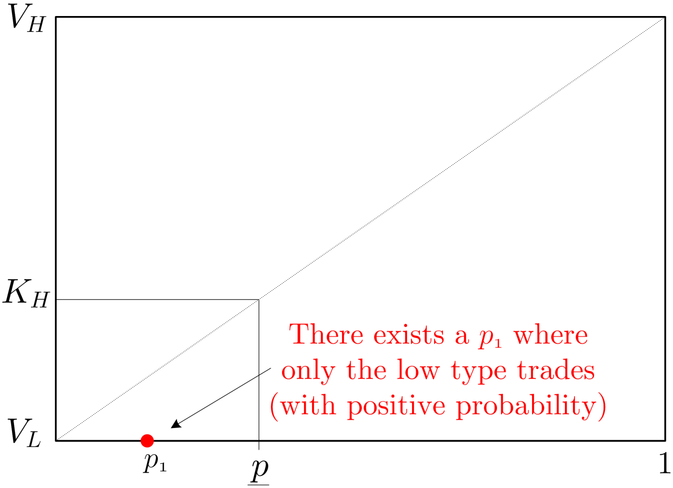

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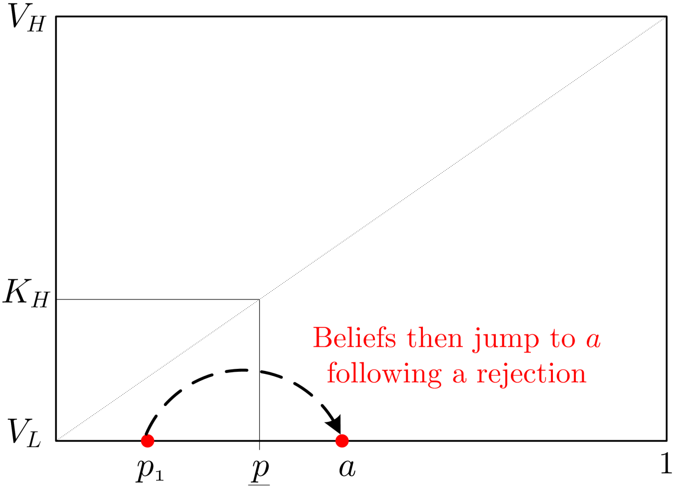

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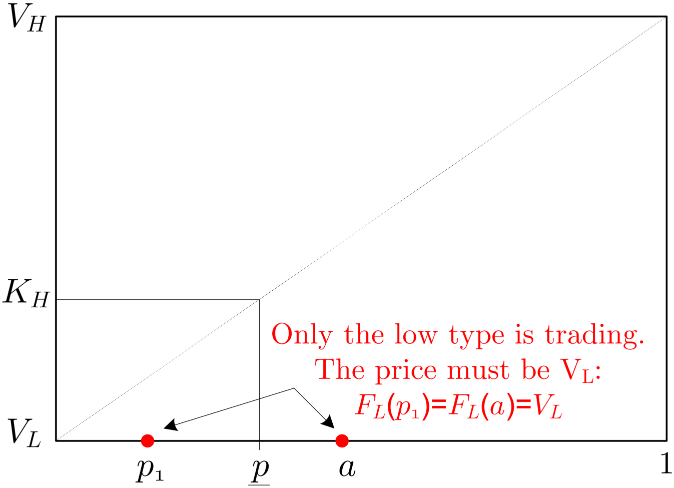

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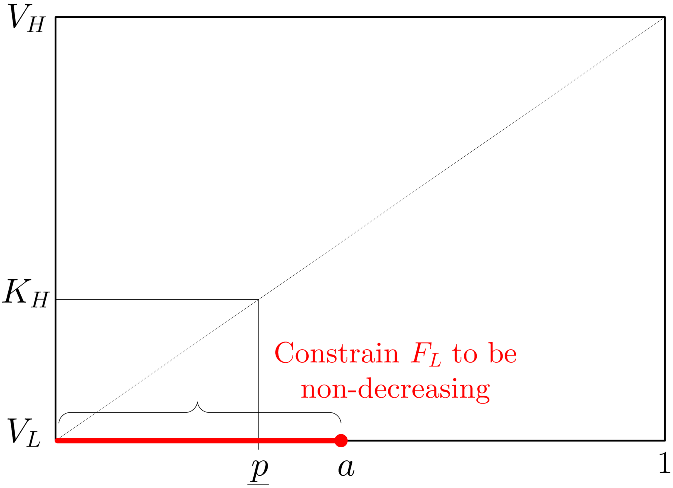

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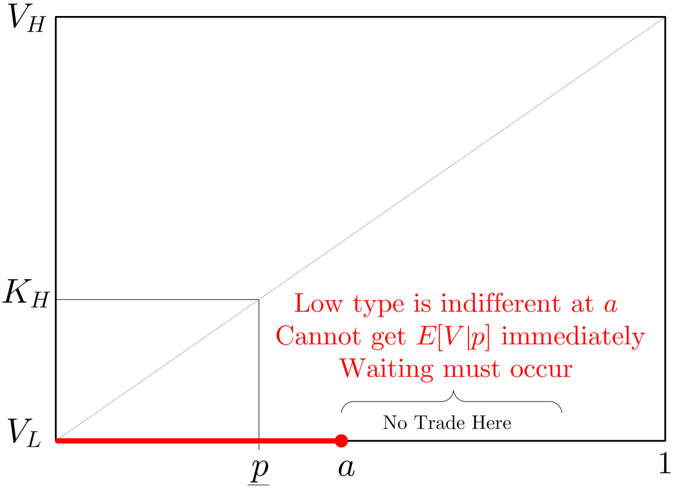

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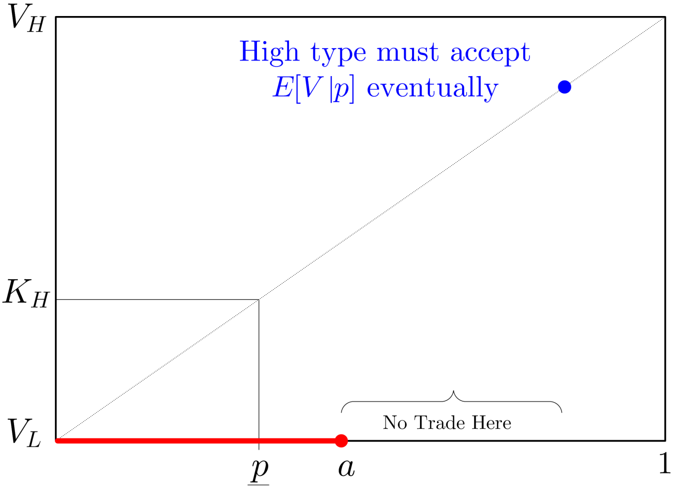

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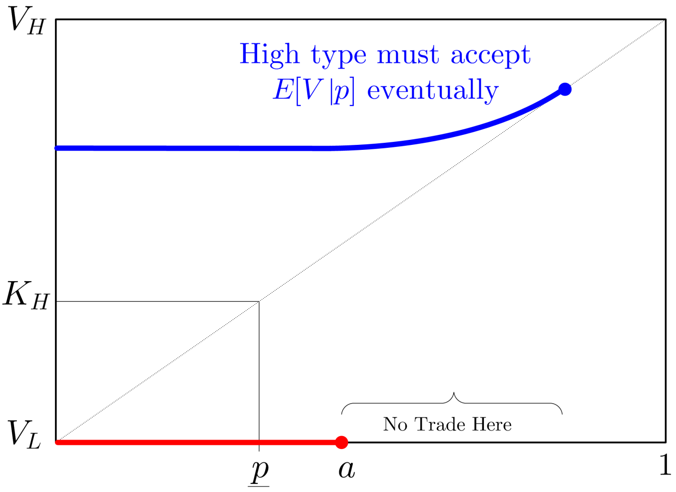

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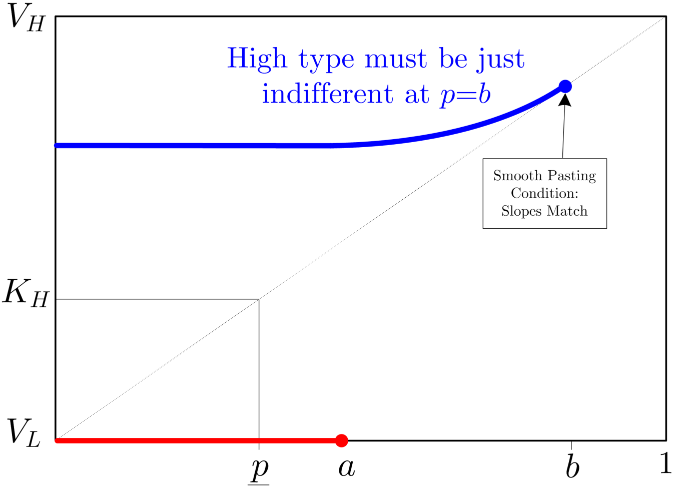

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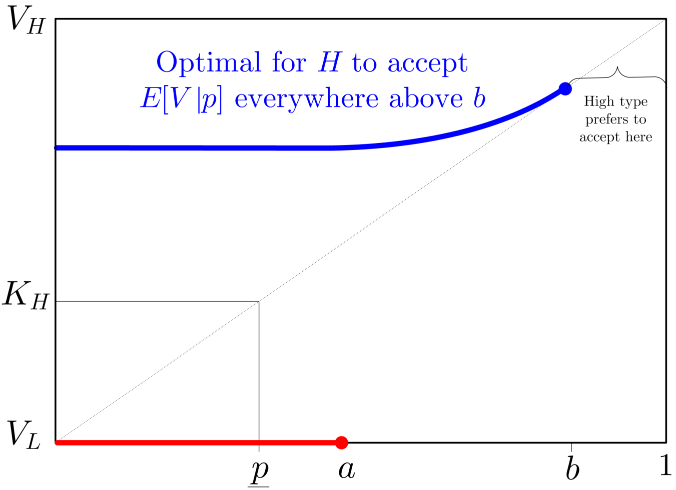

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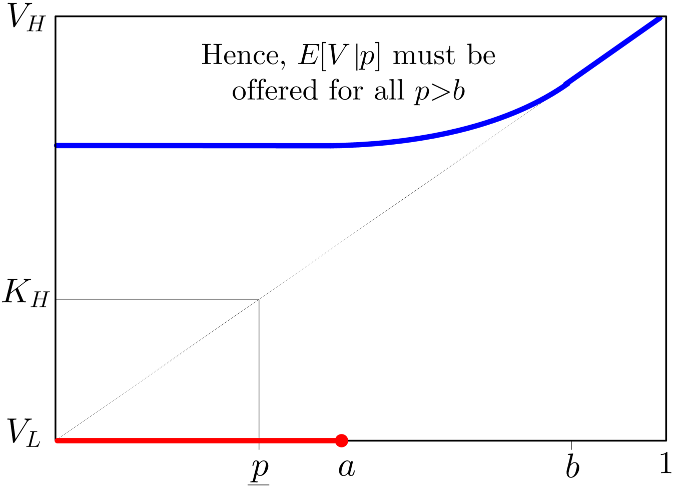

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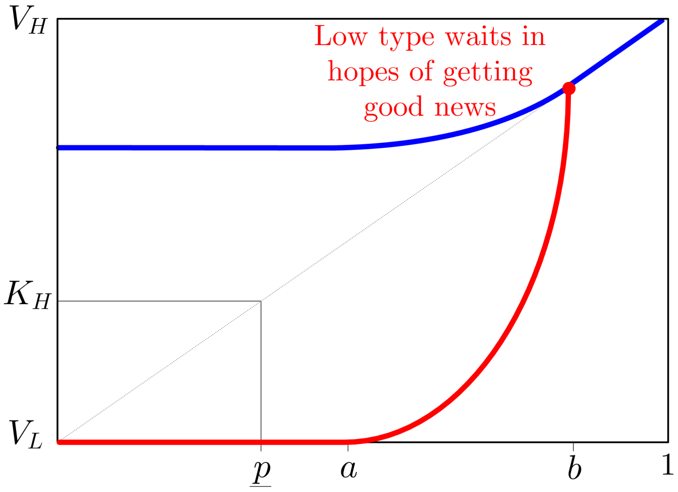

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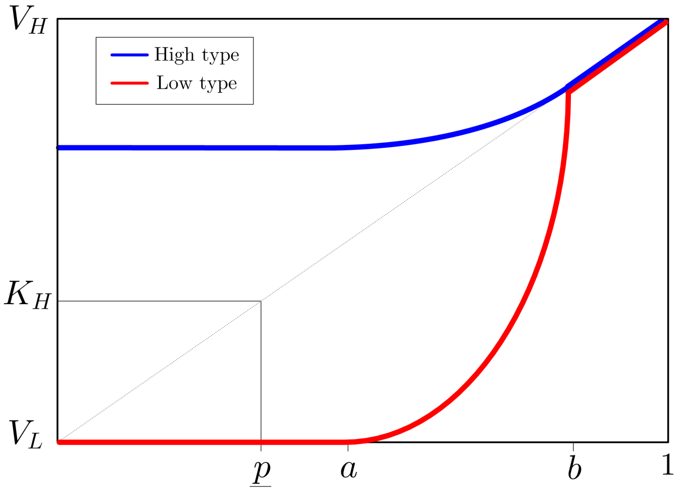


When News is Completely Uninformative

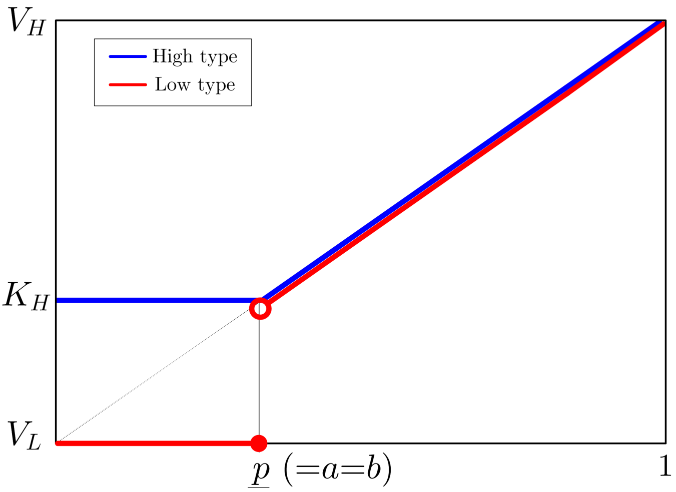
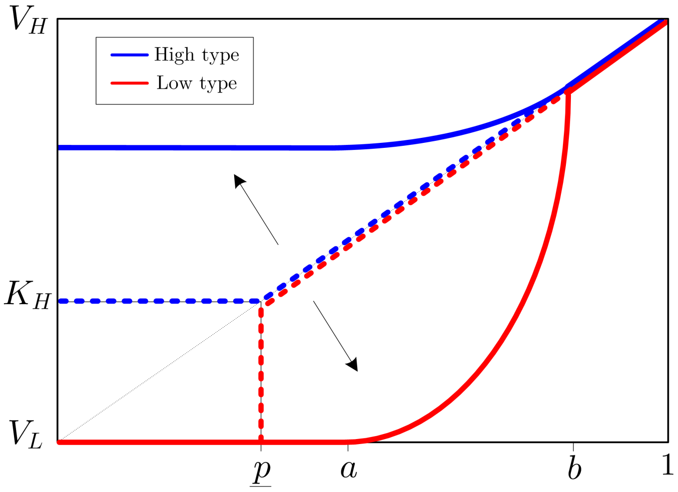


Figure : Notice, the payoffs are the same as in the static Akerlof model.

As News Becomes More Informative



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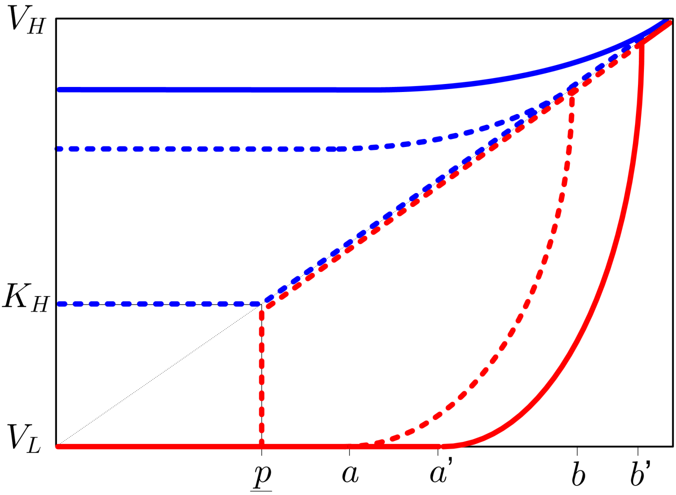


Welfare Analysis

Starting from p_0, total welfare is:

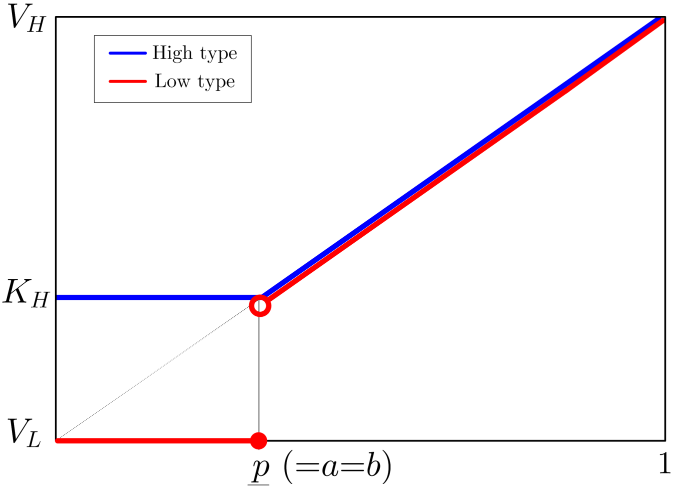
$$p_0 F_H(p_0) + (1 - p_0) F_L(p_0)$$

Total potential welfare is the expected value of the asset to buyers:

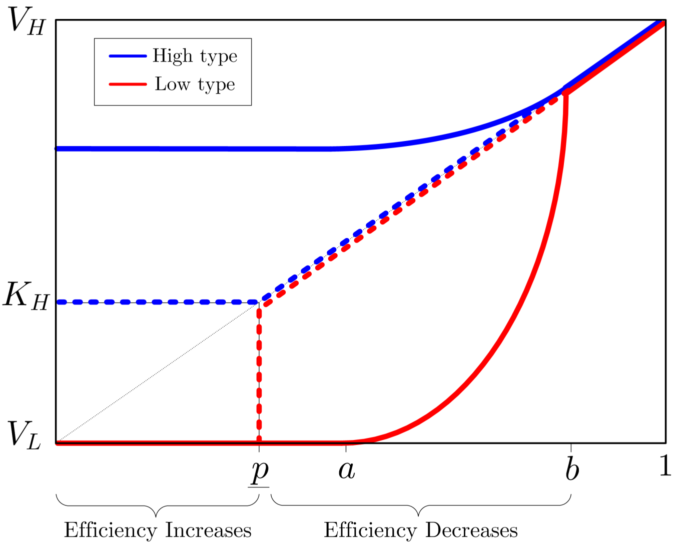
$$p_0 V_H + (1 - p_0) V_L$$

All seller's trade *eventually* so delay is the only source of inefficiency

Welfare with No News



News "Shifts" Inefficiency



Dynamic Signaling in Non-Lemons Markets

What happens when the Static Lemons Condition does not hold?

- Up to this point, we have interpreted the flow payoff to the seller as a "benefit"
- In some cases, delay may impose a "cost" to the seller
  - e.g., signaling through education or the used car dealer
- In such cases, the Static Lemons Condition may not hold
  - *That is, the high type prefers to trade at V_L rather than never trade*

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Equilibrium when $K_H < V_L$

Theorem

When the Static Lemons Condition does not hold the equilibrium depends on the quality of the news:

1. *When the news is sufficiently informative ($\phi > \underline{\phi}$), there exists an equilibrium of the same (three-region) form. Moreover it is the unique equilibrium in which the seller's value is non-decreasing in p.*
2. *When news is sufficiently uninformative, the unique equilibrium involves immediate trade at $B(z)$ for all z (similar to Swinkels, 1999).*
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2. *When news is sufficiently uninformative, the unique equilibrium involves immediate trade at $B(z)$ for all z (similar to Swinkels, 1999).*
3. *For some parameter values, there exists another type of equilibrium...*

Equilibrium when $K_H < V_L$

Theorem

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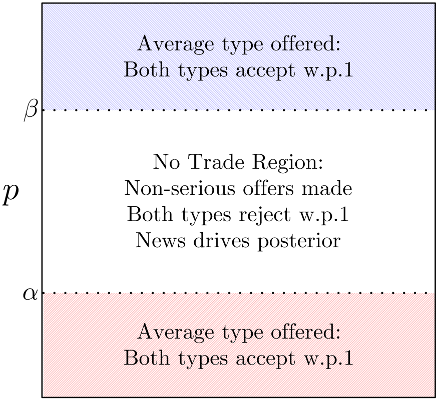
Another Type of Equilibrium

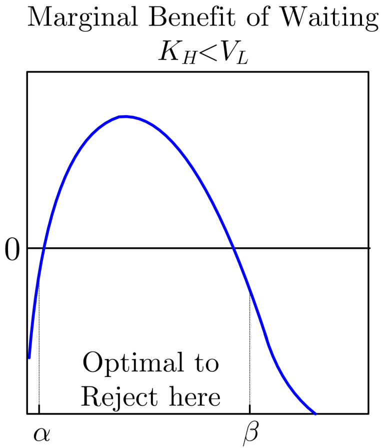
The other equilibrium looks like this:



Seller's Value in Other Equilibrium

Intuition?

- (α, β) determined solely from high type
- Low-type plays no role in determining the structure
- As *s* increases $\implies$ more incentive to wait
  - β increases
  - α decreases



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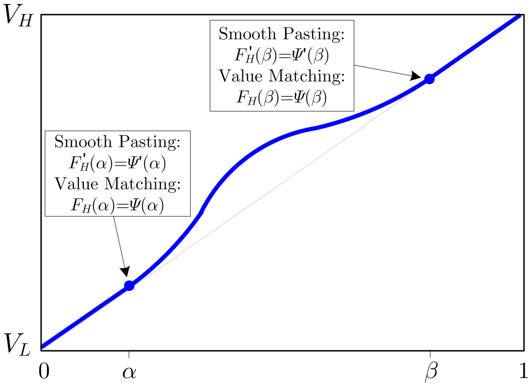

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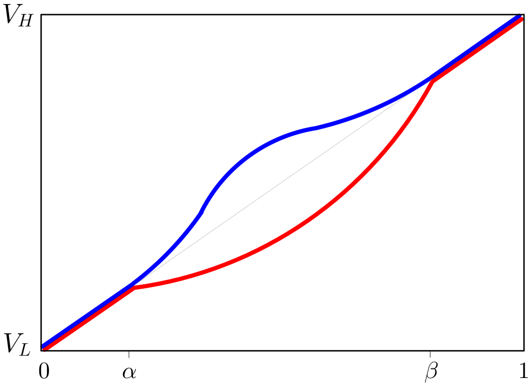

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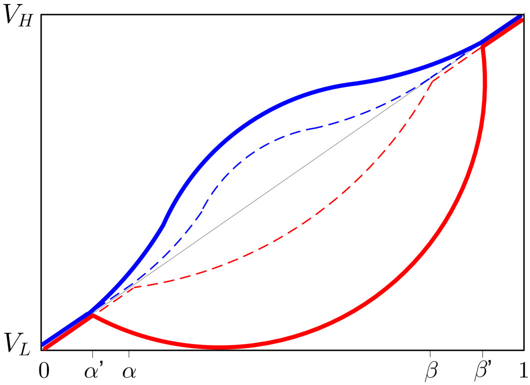

Remarks

Summary thus far...

- Introduced gradual information revelation into a dynamic lemons market
- The equilibrium involves three distinct regions: capitulation, no trade and liquid markets
- News can have a dramatic effect on trade
  - More is not necessarily better
- Developed a framework to encompass both signaling lemons markets
  - The two have the same equilibrium when news is sufficiently informative