Stockups, Stockouts, and the Role for Strategic Reserves

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Motivation

- Numerous supply disruptions have occurred in the U.S. in recent years:
  1. Gasoline shortage caused by Colonial Pipeline ransomware attack in May 2021
  2. Semiconductor chip shortage caused by the pandemic
  3. Baby formula shortage caused by the shutdown of Abbott Nutrition's plant
- Consumers response: stockpiling
  - Amaral et al (2022), Kilander (2021)

<u>This paper:</u> Price dynamics, inventory, and stockpiling during a supply disruption.

- How does consumers ability to stockpile affect firm inventory and pricing?
- Identify two forms of economic distortion
- Explore potential remedies including rationing, price controls, and strategic reserves.

Preview

1. Equilibrium Dynamics after a Disruption
   - Passive (or partial run) -> mixing
   - Firm mixes over when to hike the price
   - Fraction of full consumers increases over time

2. Economic Implications of Consumer Stockpiling
   - Hurts the firm, may hurt or harm consumers
   - Generally reduces total surplus

3. Strategic Reserves
   - Resolves underprovision of buffer stock (through quantity) and allocative distortion (through price)
   - Can achieve the social optimum

Related Literature

Self-Fulfilling Shortages

- Niedermayer (2021), Awaya and Krishna (2021), Klumpp (2021)

Consumer Stockpiling in Industrial Organization

- Guo and Villas-Boas (2007), Anton and Varma (2005), Hong et al (2002), Gangwar et al (2014)

Strategic Reserves

- Nichols and Zeckhauser (1977)

Pricing of a Storable Good with Menu Costs and Inflation

- Benabou (1989)

Model

- Monopolist firm
  - Converts an input good into output good at mc normalized to zero
  - Sells output good to consumers
- Continuum of consumers
  - Unit flow demand for the good
  - Heterogeneous values: $v_H > v_L$
  - Measure ϕ have high value, $1 - \phi$ have low value
- All players: risk-neutral, infinitely lived, discount rate $r = 0$

- There are two phases: normal and disruption
  - Normal phase: Firm can purchase input good at unit cost c
  - Disruption phase: Firm cannot purchase input good. Must rely on inventory.

- The game begins in the normal phase (at $t = 0$)
  - Disruption arrives at rate λ
  - Disruption is absorbing

Firm

- Flow inventory carrying cost $\rho > 0$ per unit
- Unlimited capacity

Consumers

- Purchase flow demand from firm if their value exceeds the price
- Can also *stock up*
  - Purchase a fixed atom of χ units
  - Flow inventory carrying cost of $\bar{\rho} \geq \rho$ per unit
- Alternative specification: smooth inventory accumulation

Parametric Assumptions

Assumption 1

$$\phi < \frac{v_L - (c + \rho/\lambda)}{v_H - (c + \rho/\lambda)} \tag{A1}$$

Two Purposes

1. Firm sells to all consumers in normal times: (A1) $\implies v_L - c > \phi(v_H - c)$
2. Firm prices at v_L at onset of disruption (without stockpiling)

Assumption 2

$$\chi \leq \bar{\chi} \equiv \frac{v_H - v_L}{\bar{\rho}} \tag{A2}$$

- Where $\bar{\chi}$ is optimal inventory level if price increase is imminent

Strategies

Firm

- Inventory level to hold in normal times, k_0
- When to raise the price (from v_L to v_H) after a disruption

Consumers

- When to stock up (if ever)

Implicit assumption: consumers draw down inventory only after price increase or firm exhausts inventory

- Optimal for small enough χ

No Stockpiling Benchmark

- k_0: buffer stock of inventory firm holds in normal state
- k_1: inventory level at which firm raises price during disruption

Firm expected profit is

$$\Pi(k_0, k_1) = \underbrace{\frac{1}{\lambda}(v_L - c - \rho k_0) - ck_0}_{\text{Normal state}} + \underbrace{v_L(k_0 - k_1) + v_H k_1 - \frac{\rho}{2}(k_0^2 - k_1^1) - \frac{\rho}{2\phi}k_1^2}_{\text{Disruption state}}$$

Result

In the no stockpiling benchmark:

$$k_0^* = \rho^{-1}(v_L - c - \rho/\lambda)$$
$$k_1^* = \frac{\phi(v_H - v_L)}{\rho(1-\phi)}$$

Planner's solution

Consider a social planner with the same production and storage technology.

- The socially optimal inventory level in the normal state is

$$k_0^{sp} = \rho^{-1} \left(\phi v_H + (1 - \phi) v_L - c - \rho/\lambda \right) > k_0^*$$

  Seller does not capture all surplus from inventory, so holds **too little**.

- The inventory level at which it is socially optimal to allocate only to high types is

$$k_1^{sp} = \frac{\phi(v_H - v_L)}{\rho} < k_1^*$$

  Seller extracts more surplus from H with price hike, so it hikes **too soon**.

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Model with Stockpiling

Preliminaries

- Equilibrium concept: MPE
  - Ties broken in consumers favor via left continuous price path
  - Akin to DT game where: 1) firm sets price, 2) state of world observed, 3) consumers choose quantity

- The firm's strategy in the benchmark is **not** an equilibrium with stockpiling
  - All H consumers would stock up right before price hike
  - Firm should hike price right before stock up

  $\implies$ Deterministic price hike will not be part of an equilibrium

Equilibrium Structure

Normal state

- Price is v_L, all consumers service their flow demand
- Firm holds inventory k_0, no stock ups

Disruption state

- Pre hike: there are four regions, not all are on path
- Post hike: Only empty H consumers service flow demand from seller
  - Full consume their stockpile until empty

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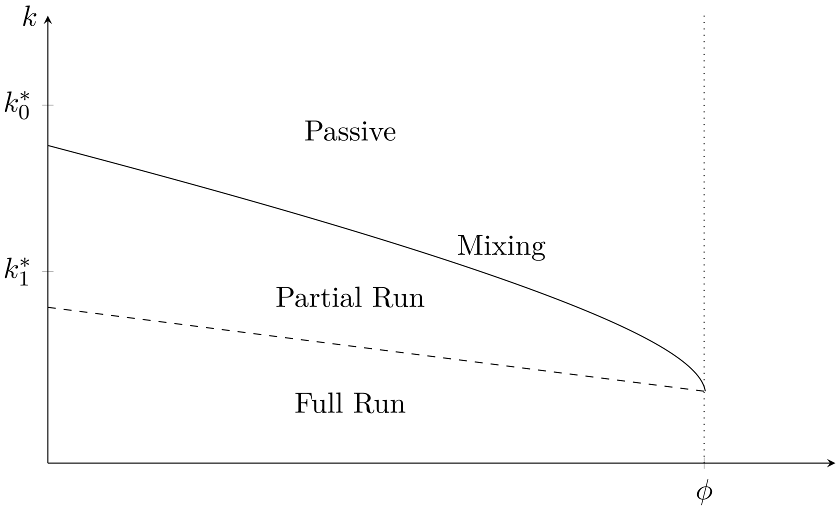
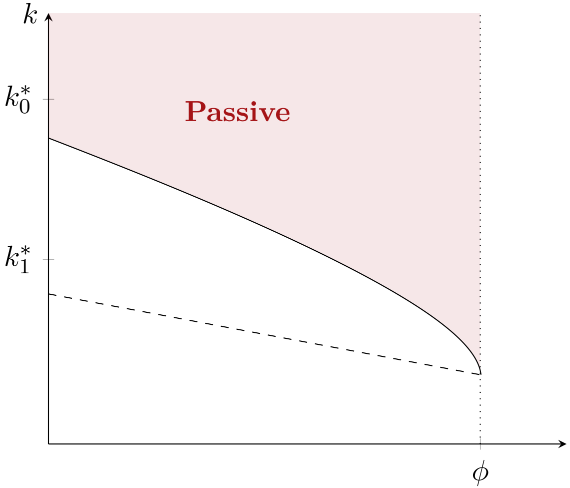
Equilibrium Structure - Disruption Subgame



Equilibrium Structure



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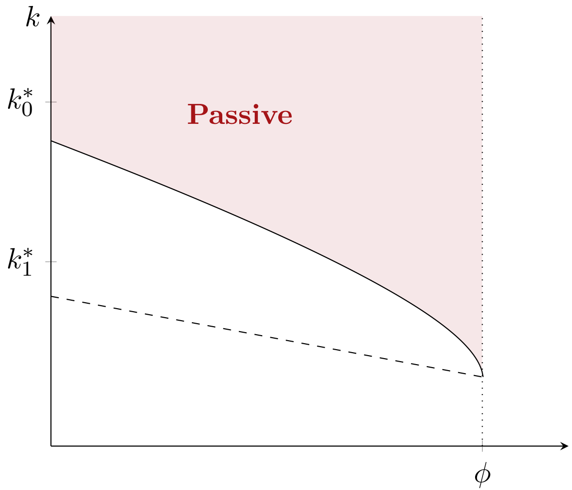


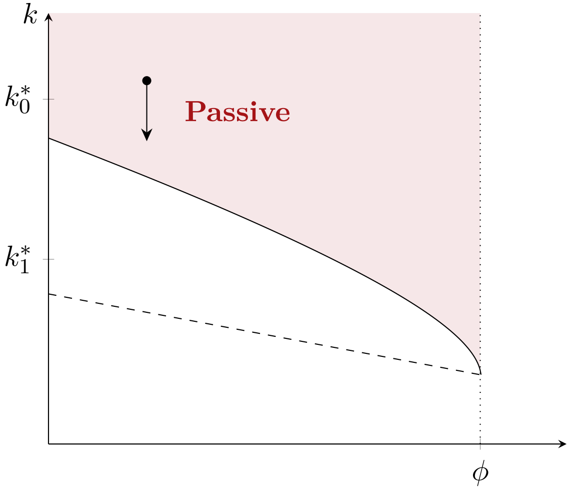
Consumers

- No one stocks up

Firm

- Keeps the price low

Equilibrium Structure



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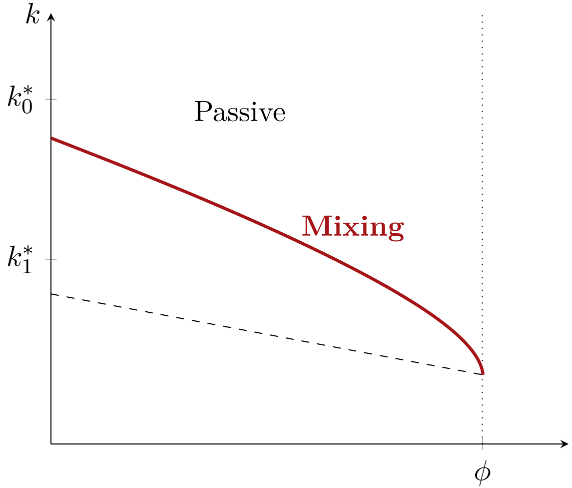


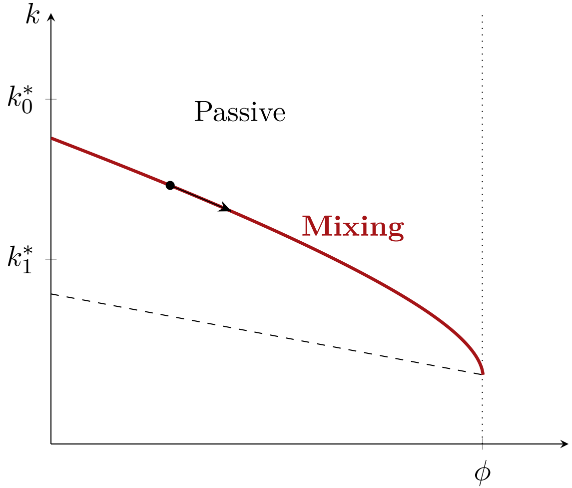
<u>Consumers</u>

- Consumers stockpile at rate μdt

<u>Firm</u>

- Hikes price with probability σdt

Equilibrium Structure



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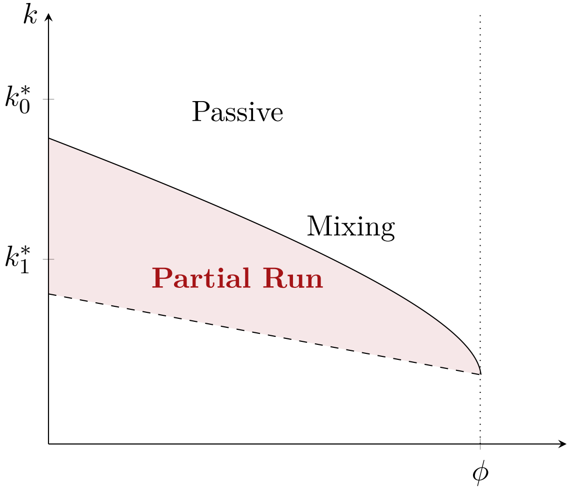
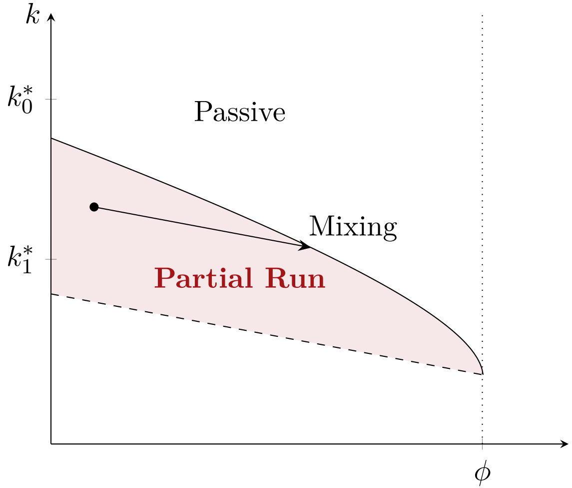


Consumers

- Mass of empty consumers stock up
- Then mixing resumes

Equilibrium Structure



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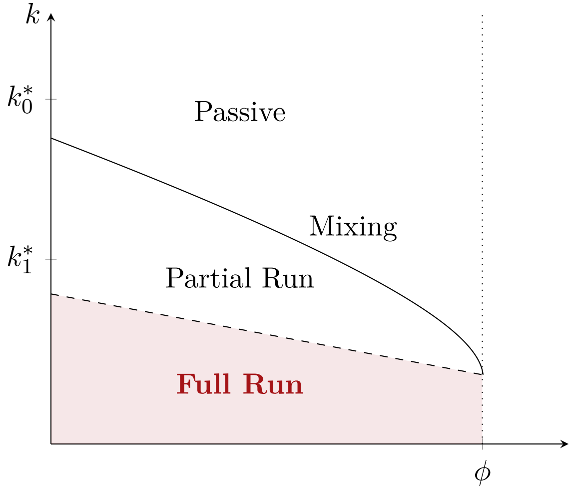


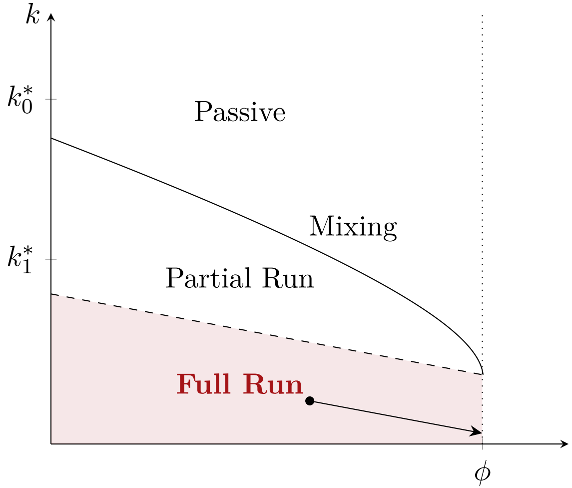
Consumers

- Empty consumers run to stock up

Firm

- Hikes the price immediately

Equilibrium Structure



Consumers

- Empty consumers run to stock up

Firm

- Hikes the price immediately

Disruption Subgame

Result

There exists a unique equilibrium of the disruption subgame, which is the one just described.

Existence Proof

Steps Involved

1. Find σ that makes empty consumers indifferent on path
2. Characterize candidate $\dot{z}(\mu)$ for firm indifference on path
   - Yields a vector field...how to select which path?
3. Equilibrium arguments to pin down the terminal point of the path
   - Pins down a unique candidate mixing path
4. Verification (omitted)

Economic Forces

- Consumer's μ makes the firm indifferent:
  - Early hike $\implies$ high k, slow depletion of k.
  - Late hike $\implies$ many full H consumers, so slow depletion of k.
- Firm's σ makes empty consumers indifferent:
  - Early stockup $\implies$ high expected carrying costs.
  - Late stockup $\implies$ price hike likely to happen before.

Pinning down σ

After a price hike:

- An empty consumer's payoff is 0
- A full consumer's payoff is $v_H\chi - \bar{\rho}\int_0^{\chi}(\chi - t)dt = v_H\chi - \frac{\bar{\rho}}{2}\chi^2$

Prior to a price hike

$$\dot{z} = \mu(\phi - z), \quad \dot{k} = -(1 + \dot{z}\chi)$$

- A full consumer's payoff is

$$f(z,k) = \overbrace{(v_H - v_L - \bar{\rho}\chi)dt}^{\text{flow}} + \sigma dt \overbrace{\left(v_H\chi - \frac{\bar{\rho}\chi^2}{2}\right)}^{\text{price hike}} + (1 - \sigma dt)\underbrace{f(z + \dot{z}dt, k - (1 + \dot{z}\chi)dt)}_{\text{no hike}}$$

Pinning down σ

HJB equation for empty consumers is

$$\sigma g = \sup_{\mu} v_H - v_L + \mu(f - v_L\chi - g) + g_z\dot{z} - g_k(1 + \dot{z}\chi) \tag{HJB-C}$$

- Necessary condition for indifference

$$g = f - v_L\chi \tag{1}$$

- Collecting the remaining terms from (HJB-C) and solving for σ, we get

$$\sigma := \frac{\bar{\rho}}{v_H - v_L - \frac{\bar{\rho}x}{2}}. \tag{2}$$

Candidate $\dot{z}$

Consumer's stockpiling strategy must make the firm indifferent

- F, firm profit after price hike (easy to compute)
- G, firm profit prior to price hike

The seller's HJB is

$$0 = \sup_{\sigma} \left\{ v_L(1+\dot{z}) + \sigma(F-G) - G_k(1+\dot{z}\chi) + G_z \dot{z} \right\}$$

- Seller's problem is linear in σ
  $\Longrightarrow$ $G = F$ is necessary in the mixing region
- Solving the remaining terms for $\dot{z}$ we get a vector field—velocity of z for each state

$$\dot{z} = \chi \frac{k - (k_1 - \frac{z}{1-\phi})}{k_1 - \frac{\phi\chi}{2(1-\phi)}}$$

Equilibrium Mixing Path

- Of course, z and k are linked in equilibrium.
- Before the price hike
  - z rises at rate $\dot{z}$
  - k falls at rate $1 + \dot{z}$
- So the equilibrium mixing path $\hat{k}(z)$ is given by:

$$\frac{dk}{dz} = -\frac{1 + \dot{z}}{\dot{z}}$$

Useful shorthand notation:

- $\alpha := \frac{\phi}{1-\phi}$
- $\underline{k} := k_1^* - \alpha\chi$
- $\underline{\sigma} := \frac{\rho}{v_H - v_L - \frac{\rho\chi}{2}}$

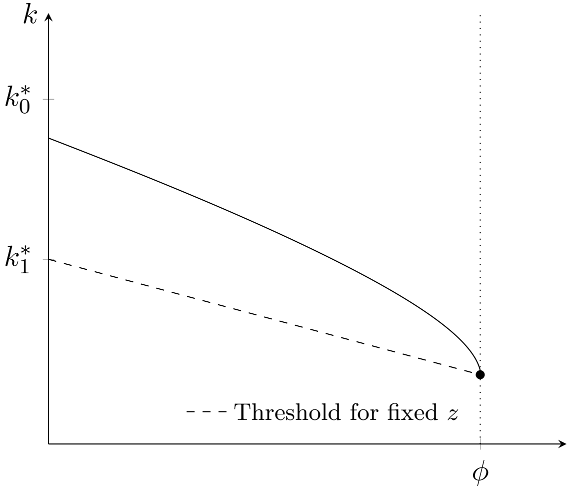
Terminal Condition?

Lemma 1

The mixing path must end at $(z, k) = (\phi, \underline{k})$, at which point all consumers are full and the firm raises the price with probability one.

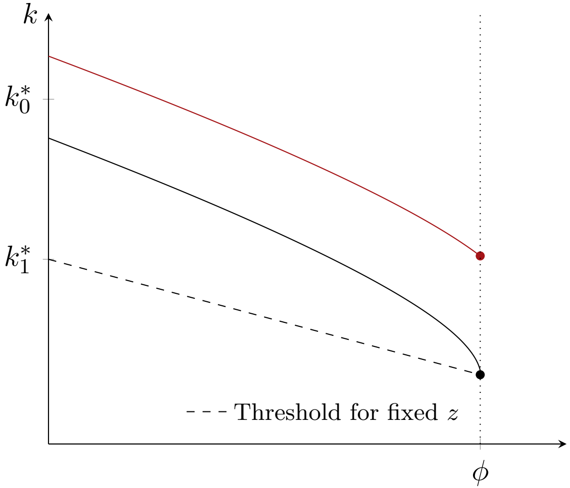
- Price mixing must end weakly before all consumers are full.
- But if any consumers are empty after mixing ends, they would have been better off stocking up sooner.
- It follows that all consumers are full just when mixing ends.

Illustration of Lemma 1



- Why is the path unique?

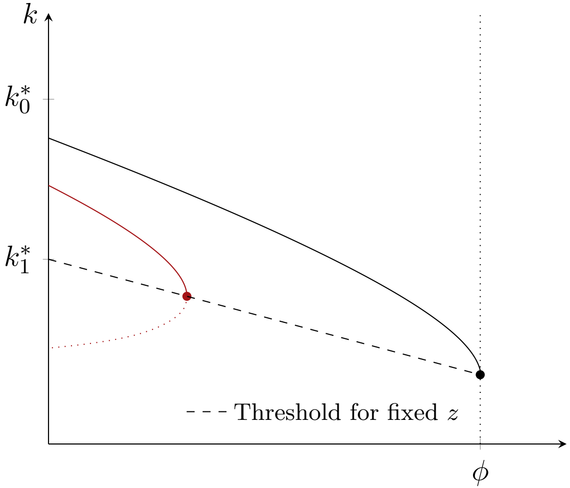
Illustration of Lemma 1



Above the equilibrium path?

- Eventually $z = \phi$
  - $\dot{z} = 0$ thereafter
- But then firm wants to keep the price low
- Recent stock ups not optimal

Illustration of Lemma 1



Below the equilibrium path?

- Firm hikes price when path hits threshold
- Empty consumers should stock up beforehand

The Equilibrium Mixing Path

Solving (ODE) with the terminal condition in Lemma 1 yields the following.

Lemma 2

The unique equilibrium mixing path in the disruption phase, $\hat{k}(z)$, is given by

$$\hat{k}(z) = k_1^* - \frac{z}{1-\phi} + \frac{1}{\underline{\sigma}}\left(1 + W\left(-e^{-1-\alpha\underline{\sigma}(\phi x - z)}\right)\right) \tag{MP}$$

It starts at $\overline{k} \equiv \hat{k}(0)$ and ends at $\underline{k}$ when all consumers are full.

- W is the Lambert (aka product log) function implicitly defined by

$$W(y)e^{W(y))} = y$$

Equilibrium in the Normal Phase

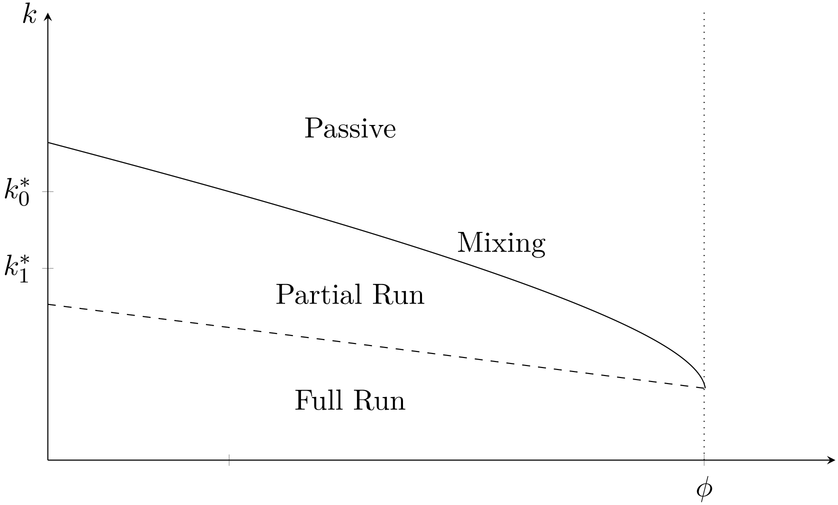
- Consumers act first when the disruption hits (simpler than alternative).
  - Assumption is inconsequential in the “regular" case (i.e., (i) below)
  - Implies that consumers do not carry inventory in the normal phase.
- So the only decision is how much inventory the firm should procure.
- For $k_0^* < \overline{k}$, define $z^* := \hat{k}^{-1}(k_0^*)$.

Result

(i) *If $k_0^* \geq \overline{k}$, then the firm procures k_0^*. When the disruption hits, price remains low until mixing path is reached at $(0, \overline{k})$.*

(ii) *If $k_0^* < \overline{k}$, then the firm procures $k_0^* + z^*$ When the disruption hits there is a partial run such that the mixing path is reached immediately at (z^*, k_0^*).*

Partial Run Case



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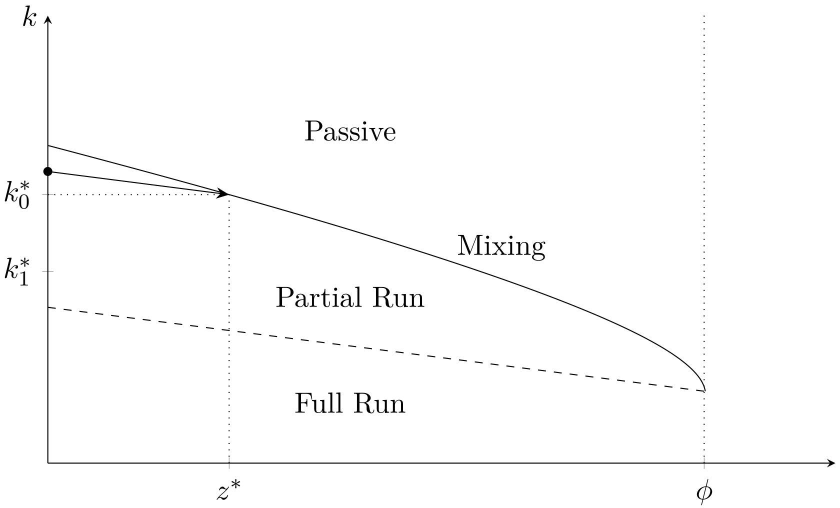

Duration of the Mixing Path

Let T be the maximum amount of time the price can remain low along the mixing path $\hat{k}(z)$.

$$T := \begin{cases} \overline{k} - (\underline{k} + \phi\chi) & k_0^* \geq \overline{k} \\ k_0^* + z^* - (\underline{k} + \phi\chi) & k_0^* < \overline{k} \end{cases}$$

Let τ be the (random) amount of time from the onset of the disruption until the firm raises the price.

$$\mathbb{E}[\tau] = \begin{cases} k_0^* - \overline{k} + \frac{1}{\sigma}(1 - e^{-\sigma T}) & k_0^* \geq \overline{k} \\ \frac{1}{\sigma}(1 - e^{-\sigma T}) & k_0^* < \overline{k} \end{cases}$$

Consumer Welfare

Benchmark consumer surplus (with no storage) is

$$U_n = (v_H - v_L)(k_0^* - k_1^*).$$

Because a consumer is indifferent about stocking up at any point on the mixing path, her expected payoff in the disruption phase can be calculated as if she planned to stock up at T:

$$U_s := (v_H - v_L)\mathbb{E}[\tau] + \left((v_H - v_L)\chi - \frac{\bar{\rho}\chi^2}{2}\right) e^{-\sigma T}$$

Result

Consumer surplus, U_s, is decreasing in $\bar{\rho}$, and there exists $\bar{\rho}_0 > \rho$ such that

$$\bar{\rho} \lesseqqgtr \bar{\rho}_0 \implies U_s \gtreqqless U_n.$$

Firm Profit and Total Surplus

- The firm is always worse off when consumers can stockpile, $\Pi_s < \Pi_n$.
  - Threat of stockpiling induces firm to raise the price sooner (in expectation) than optimal
- For $i \in \{n, s\}$, define social welfare by

$$\Omega_i := \Pi_i + \phi U_i$$

Result

(i) $\Omega_s < \Omega_n$ *if* $\bar{\rho} > \bar{\rho}_0$.

(ii) $\Omega_s < \Omega_n$ *if* χ *is sufficiently small.*

(iii) $\Omega_s > \Omega_n$ *iff* χ *is not too small or too large and* $k_0^* - k_1^*$ *is sufficiently small.*

Policy Implications

- Rationing
  - Restricting stockpiling is welfare improving (for some parameters)
  - But does not resolve benchmark distortions
  - Informal market can arise where L sell to H
- Price Controls
  - Prevents efficient allocation
  - Produces a run when $k = \phi\chi$
  - Does not solve underprovision of inventory
- Strategic Reserves
  - Our focus

Strategic Reserves

- The US government maintains stockpiles of various goods that can be drawn upon during supply disruptions
  - Examples: medical equipment, petroleum, rare earth metals, helium, cheese
- Stated intention is for national security
  - Also subsidizes farmers
- Question: is there an economic role for strategic reserves (even absent national security considerations)?

Strategic Reserves

- We introduce a third player (the government, G)
  - Objective is to maximize total welfare
  - Can hold/release strategic reserves (of the output good)
  - Same inventory costs as the firm, ρ

Definition

*A **simple policy** is one where the government holds some amount Q of strategic reserves in the normal state and commits to sell them for a price of v_L during a disruption.*

- Response: firm hikes price at $k_1(Q)$, $\downarrow$ in Q
  - k_0 is independent of Q
  - marginal unit sells immediately for v_L

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Strategic Reserves in the No Stockpiling Benchmark

Recall the two distortions in the no stockpiling benchmark

- Firm holds too little inventory
- Firm increases the price too soon

Result

In the no stockpiling benchmark, the government can achieve the social optimum using a simple policy with $Q^* = \frac{\phi(v_H - v_L)}{\rho}$.

Two distortions, one degree of freedom. Coincidence?

Strategic Reserves with Consumer Stockpiling

Result

With consumer stockpiling and $\bar{\rho} > \rho$, the government cannot achieve the social optimum with a simple policy.

- However, G can increase welfare with simple policy...
  - Optimal Q is same as in no stockpiling benchmark
  - Inefficiency? a simple policy induces a “run on reserves"
    - Consumers stockpile when reserves are running low (inefficient)
    - Takes longer to exhaust reserves
    - Delays price hike: $k_1(Q^*) < k_1^*$
  - Fails to achieve social optimum

Strategic Reserves with Consumer Stockpiling

Result

With consumer stockpiling, the government can achieve the social optimum by holding Q^* in reserves and charging a price that increases as reserves are depleted.

- Holding Q^* in reserves, only to be sold to high types, is sufficient to prevent firm from a price hike
- Once firm inventory is exhausted, release reserves at a price (starting from v_L) that **gradually** increases as reserves are depleted
- Gradual price increase makes consumer indifferent to stockpiling because they will continue to service flow from reserves for some time so inventory costs are high

Conclusion

- Investigate price dynamics, inventory choice, and consumer stockpiling in the face of a supply disruption.
- Two economic distortions
  - Under provision of buffer stock inventory
  - Inefficient allocation of inventory
- Consumer stockpiling hurts the firm, may or may not benefit consumers, and generally reduces total surplus
- Strategic reserves can restore the social optimum