Market Signaling with Grades

Brendan Daley & Brett Green

Introduction

- Traditional Signaling Models: A privately informed sender can convey his information only through costly action.
- Least-cost separation is the only "stable" equilibrium. (Cho & Kreps 1987).
- Questionable Predictions:
  - Equilibrium does not depend on prior beliefs.
  - High signaling costs even if receivers are virtually certain the sender is the high type.
  - Equilibrium does not depend on high type's costs.
  - No role for external information.
- Example: Elantra vs. Civic warranties
- Adding grades resolves all of these issues.

What's Missing?

- In many signaling environments, additional information is also available.
- We enrich the standard signaling model by introducing *grades*.

Grades

- A **signal** is a costly action the sender chooses.
- A **grade** is a free public message about the sender's type.
  - School grades
  - Product and service reviews
  - Bond ratings
  - Auditor reports
- Grade accuracy may or may not depend on the costly signal.
- We characterize how the two channels of information transmission interact in a strategic environment.

Equilibrium with Grades

With informative grades, in the unique “stable” equilibrium:

1. The high type resolves his tradeoff between relying on the costly signal and relying on the grade.

    Equilbrium depends on the high type’s *relative* cost advantage and *relative* grade advantage.

2. There is at least partial pooling. Hence, grades convey meaningful information & affect outcomes.

Equilibrium with Grades

3. If the prior puts sufficient weight on the high type, the equilibrium is full pooling.

4. For both types, signaling costs are decreasing and utilities are increasing in the prior.

5. If the grade is always sufficiently informative, the equilibrium converges to the full information outcome, involving no signaling, as the prior's weight on the high type approaches 1.

Traditional Market Signaling

- There is one sender and multiple receivers.
- The sender is privately informed about his type θ: H or L.
- The receivers share a commonly known prior $\mu_0 \in (0, 1)$ that the sender is the high type.
- The sender chooses a signal $x \in \mathbb{R}_+$ and incurs cost $C_\theta \cdot x$.

$$C_L = 1 > C_H = 3/4$$

The Market

- After observing the signal, x, the receivers update their beliefs from μ_0 to $\mu(x)$.
- Each receiver i, simultaneously offers a wage/price, $w_i(x)$.
- The sender decides which offer (if any) to accept.

Payoffs

- Sender:

$$\begin{aligned} w - C_\theta \cdot x & \quad \text{if trades} \\ - C_\theta \cdot x & \quad \text{if not} \end{aligned}$$

- Receiver:

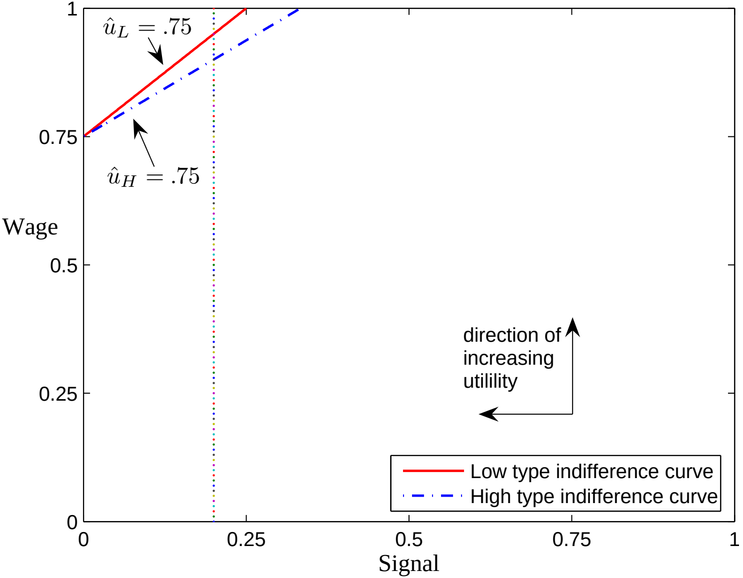
$$\begin{aligned} 1-w & \quad \text{if trades with high type} \\ -w & \quad \text{if trades with low type} \\ 0 & \quad \text{if does not trade} \end{aligned}$$

- The receivers engage in Bertrand Competition. Therefore, the sender is always paid his expected value.

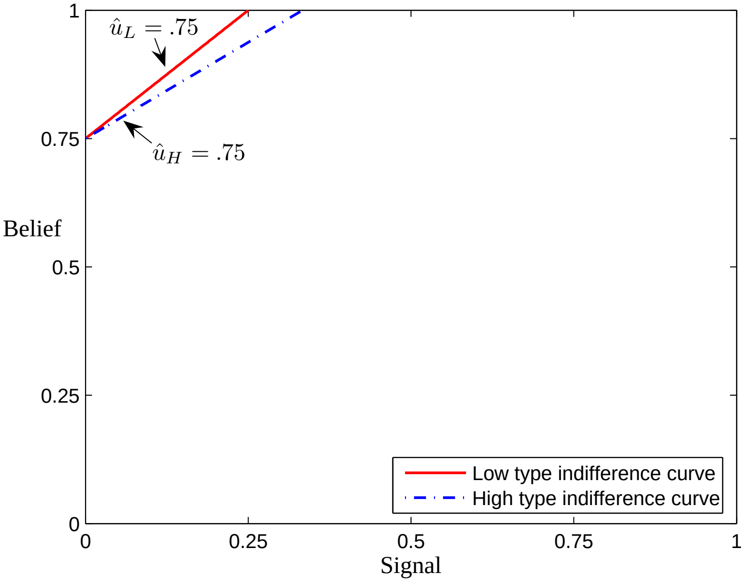
Potential Equilibria

- Let $\mu_0 = \frac{3}{4}$.
- Full Pooling Equilibrium: Both types select $x = 0$.
  $u_H^* = u_L^* = .75$
- Least Cost Separating Equilibrium: Low type selects $x = 0$, High type selects $x = 1$. $u_H^* = .25, u_L^* = 0$
- Pooling Equilibrium fails refinement D1.
- The LCSE is the unique equilibrium satisfying D1.

Indifference Curves



Indifference Curves



D1

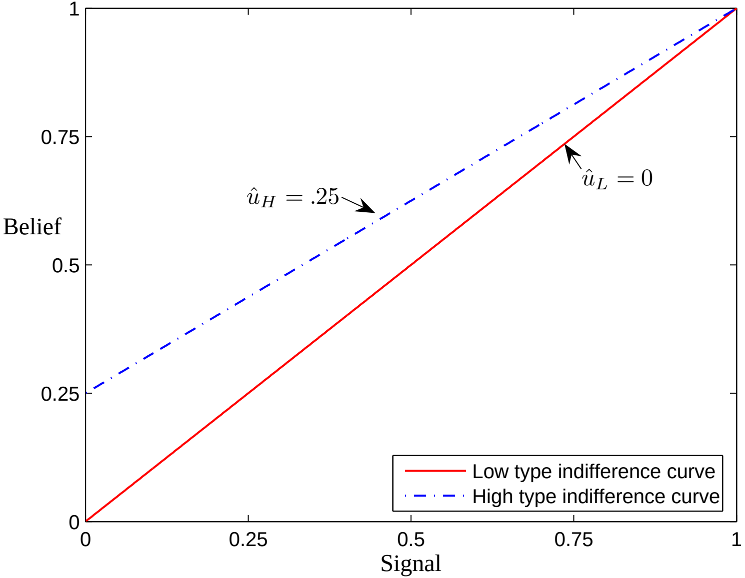
- The D1 Refinement: (Banks and Sobel 1987)

  Fix an equilibrium yielding utilities $\{u_H^*, u_L^*\}$.
  Let $B_\theta(x, u_\theta^*)$ be the set of beliefs μ such that

  $$u_\theta(x, \mu) > u_\theta^*$$

  To satisfy D1, for any x that is not on the equilibrium path,
  - if $B_H(x, u_H^*) \subset B_L(x, u_L^*)$, then $\mu^*(x) = 0$.
  - if $B_L(x, u_L^*) \subset B_H(x, u_H^*)$, then $\mu^*(x) = 1$.

Least Cost Separation



Grades

- Introduce an exogenous binary grade: $g \in \{g_L, g_H\}$.
- Probability that type θ obtains g_θ is $p = \frac{3}{4}$.

The Market

- The sender selects a signal x.
- All players observe x and g.
- Receivers update their beliefs from μ_0 to $\mu(x)$. We call μ the receivers' *interim* belief.
- Receivers update from $\mu(x)$ to a final belief based on g via Bayes Rule.
- Each receiver i simultaneously offers a wage $w_i(g, x)$.
- Again, the sender will be paid his expected value based on the receivers final beliefs.

Interim Beliefs

- Every interim belief $\mu(x)$ pins down a wage for each grade:

$$\begin{aligned} w(g_L, x) &= \frac{\mu(x)}{\mu(x) + (1-\mu(x))\frac{p}{1-p}} \\ w(g_H, x) &= \frac{\mu(x)}{\mu(x) + (1-\mu(x))\frac{1-p}{p}} \end{aligned}$$

- And therefore expected utilities for both types:

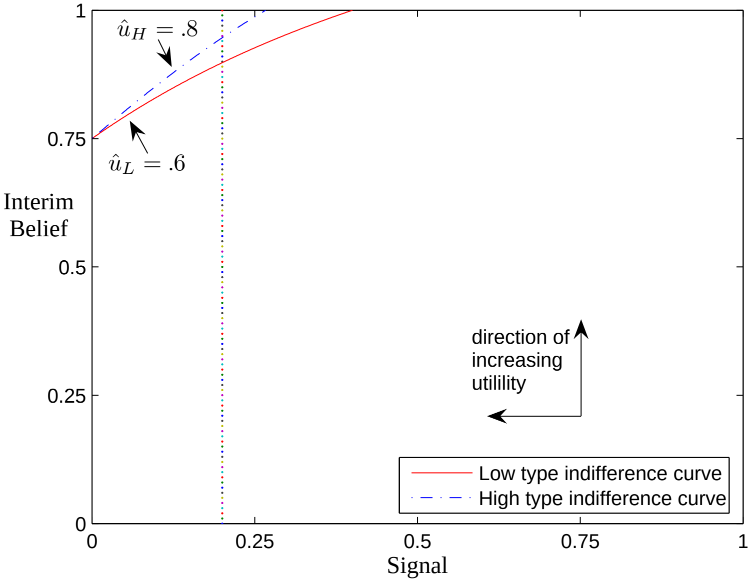
$$u(x, \mu(x)) = p \cdot w(g_\theta, x) + (1-p)w(g_{\theta'}, x) - C_\theta \cdot x$$

- We can draw indifference curves using expected utilities over (x, μ).

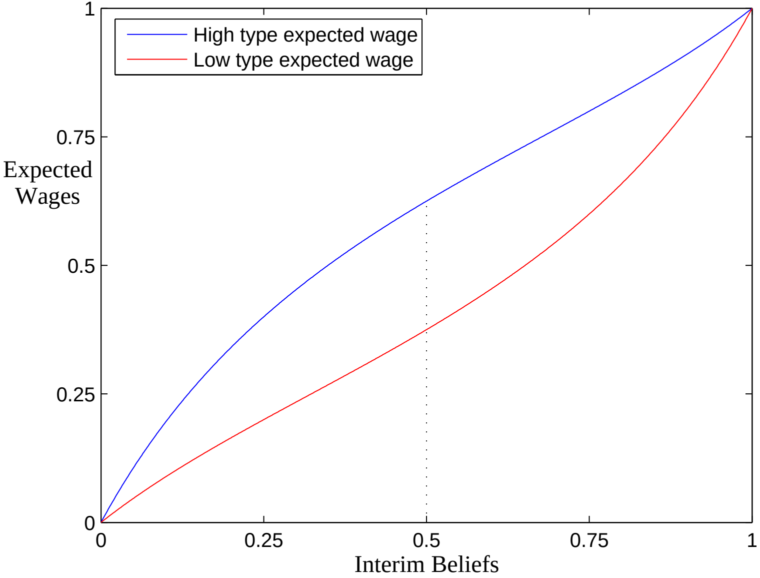
Potential Equilibria

- Let $\mu_0 = \frac{3}{4}$.
- Full Pooling: Both types select $x = 0$. $u_H^* = .8, u_L^* = .6$
- LCSE: Low type selects $x = 0$, High type selects $x = 1$. $u_H^* = .25, u_L^* = 0$
- Full pooling at $x = 0$ survives D1.
- LCSE fails D1.
- Full pooling at $x = 0$ is the **unique** equilibrium satisfying D1.

Full Pooling Survives



The Shape of Indifference Curves



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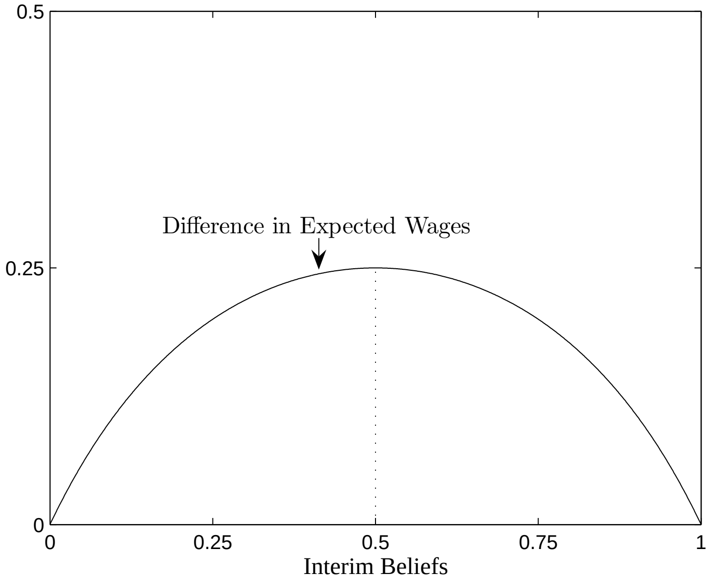

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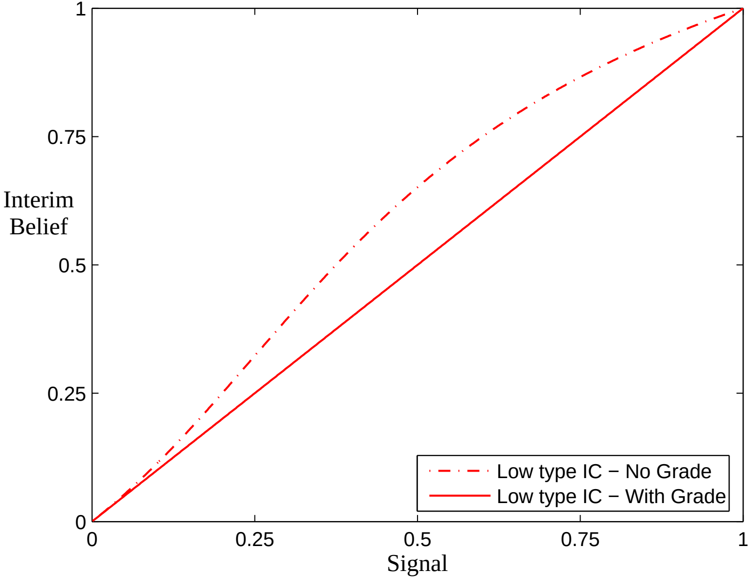

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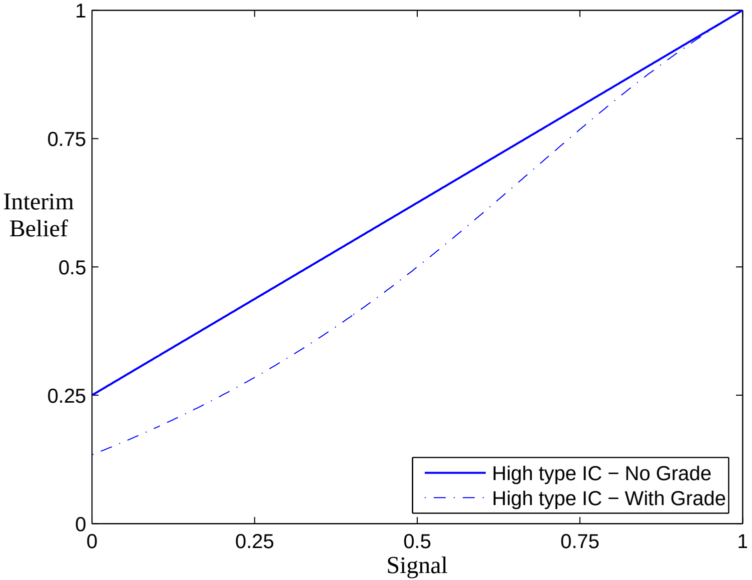
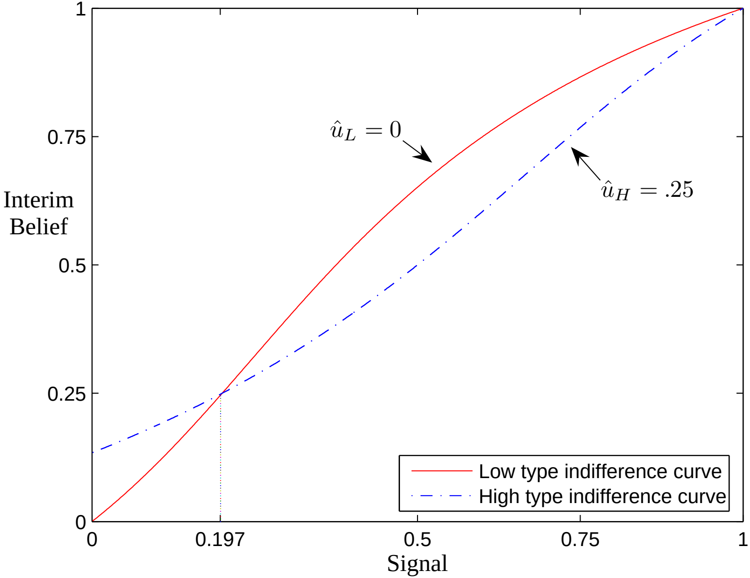


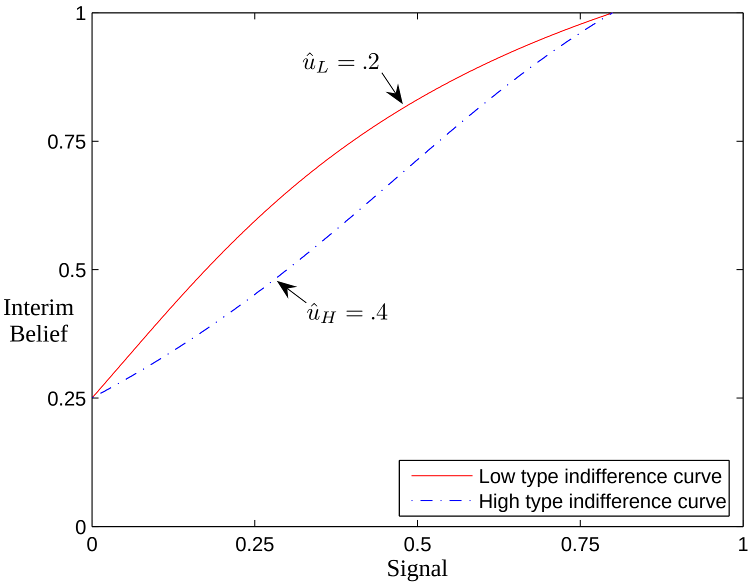
LCSE Fails D1



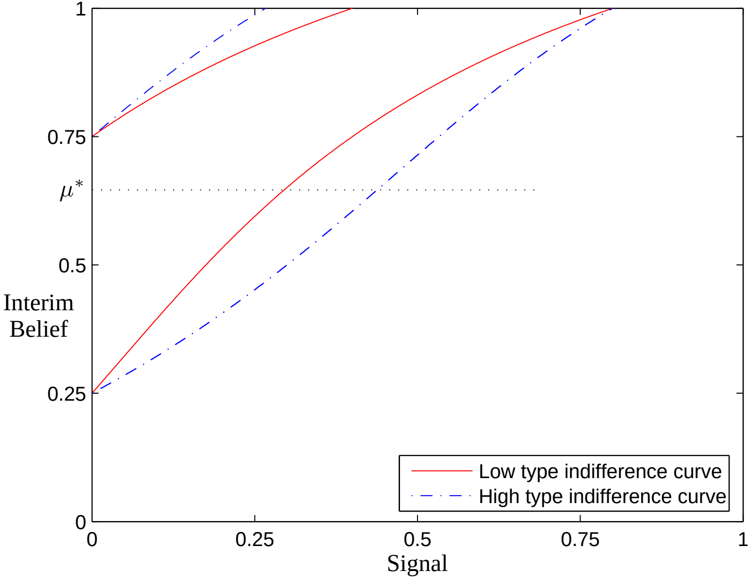
The Equilibrium Depends on the Prior

- Let $\mu_0 = \frac{1}{4}$.
- Recall that in standard signaling, the unique D1 equilibrium, the LCSE, does not depend on the prior.
- When $\mu_0 = .75$, with the grade, the unique D1 equilibrium was full pooling.
- Potential Equilibrium - Full Pooling: Both types select $x = 0$. $u_H^* = .4, u_L^* = .2$.

Indifference Curves with Grades



Indifference Curves with Grades



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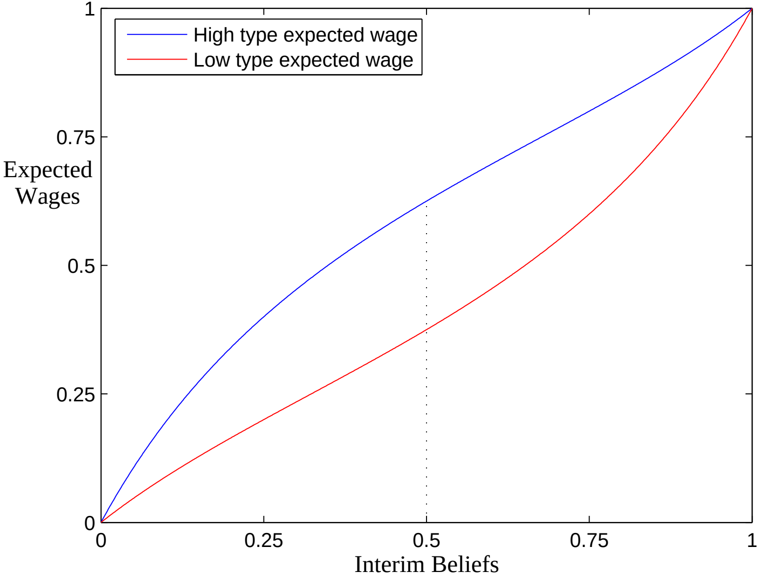

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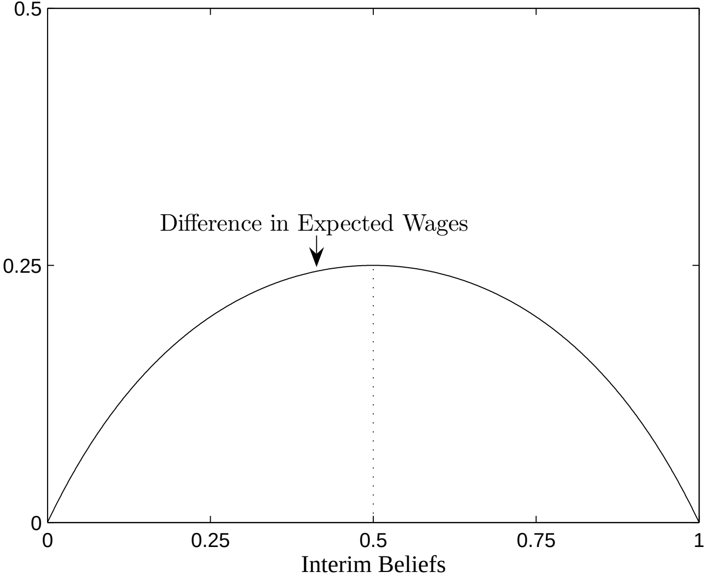


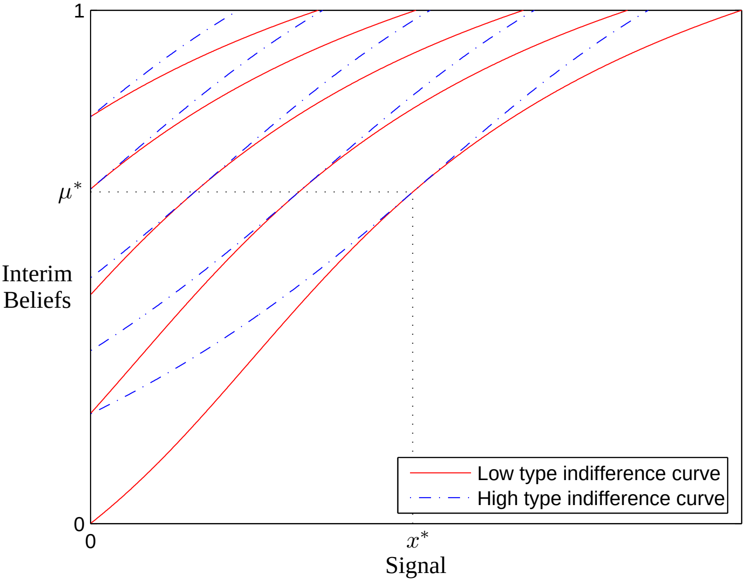
Equilibrium with Grades

For any $\mu_0 \neq \mu^*$, there is a unique equilibrium satisfying D1.

- If $\mu_0 < \mu^*$, the equilibrium is partial pooling.
    - The high type selects x^*.
    - The low type mixes over $x = 0$ and x^*.
- If $\mu_0 > \mu^*$, the equilibrium is full pooling at $x = 0$.

For $\mu_0 = \mu^*$, all equilibrium are full pooling. The pooling can be at any x in $[0, x^*]$.

Equilibrium with Grades



Grades

- Set of grades $\mathbb{R}$.
- For each type there is a pdf on $\mathbb{R}$: $\pi_\theta(g|x)$.
- $\pi_\theta(g|x)$ is differentiable in x everywhere and in g almost everywhere.
- $\pi_H(g|x) = 0$ if and only if $\pi_L(g|x) = 0$.

Informativeness Assumptions

Likelihood Ratio: $R(g|x) = \frac{\pi_L(g|x)}{\pi_H(g|x)}$

A1. For all $x > 0$, there exists a positive measure of grades such that $R(g|x) \neq 1$.

A2. For all μ in $(0, 1)$,

$$E\left[\frac{\mu}{\mu + (1-\mu)R(g|x)}|\theta = L\right]$$

is weakly decreasing in x.

A3. $E[R(g|x)|\theta = L]$ is weakly increasing in x.

Grade Examples

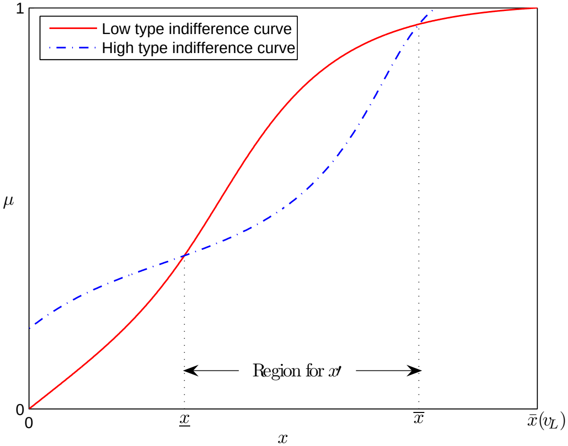
- In the example, grade technology was *binary* and *independent of x*.
- Technology that is independent of x is easiest to analyze.

Preliminary

- Let $v_H > v_L \geq 0$.
- Let $C_L \geq C_H > 0$.
- Receivers compete in Betrand Competition. The sender will be paid his expected value based on the receivers final beliefs.
- We look at indifference curves over (x, μ).

Lemma 1

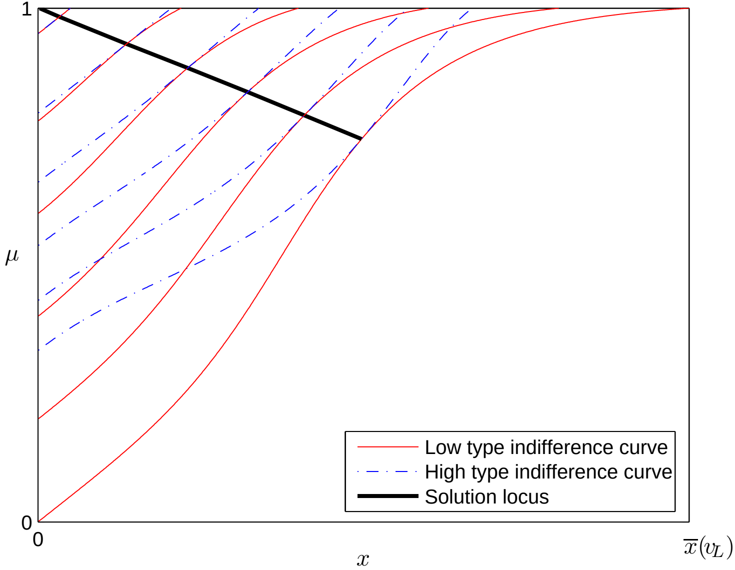
Lemma: Fix a $\{\hat{u}_H, \hat{u}_L\}$. If $\exists\ x'$ such that the high type's indifference curve for $\hat{u}_H$ is below the low type's indifference curve for $\hat{u}_L$ at x', then the payoffs $\{\hat{u}_H, \hat{u}_L\}$ are not supported by any D1 equilibrium.



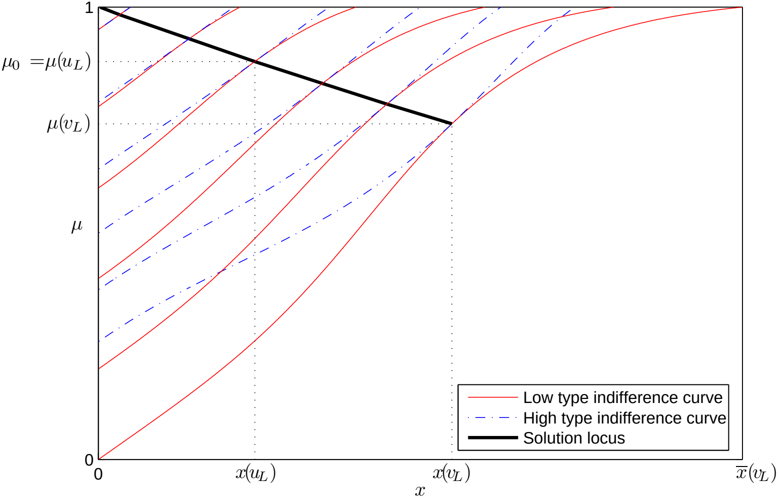
Crossing Condition

The Crossing Condition

Solution Locus



Equilibrium Picture



Equilibrium Theorem

Theorem

If $\mu(u_L)$ is strictly increasing whenever $\mu(u_L) < 1$, then there exists a unique D1 equilibrium.

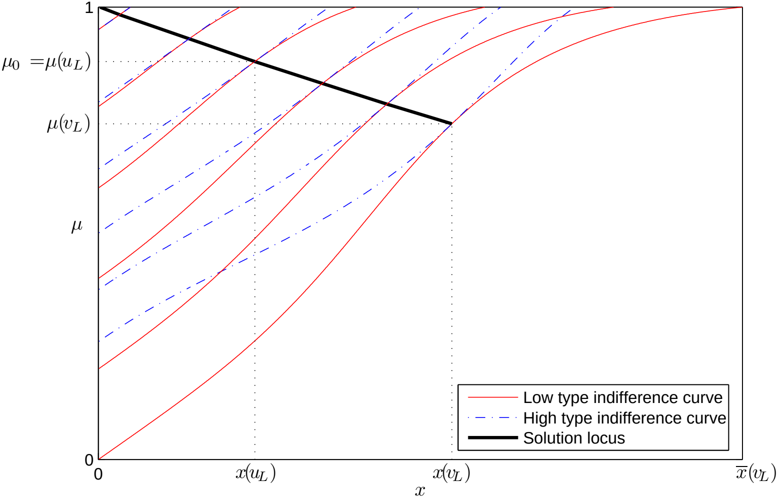
The equilibrium may be partial pooling, full pooling or fully separating depending on parameters.

- *If $\mu(v_L) = 1$, then LCSE.*
- *If $\mu(v_L) < 1$, then:*

  for $\mu_0 < \mu(v_L)$ the equilibrium is partial pooling.

  for $\mu_0 \geq \mu(v_L)$ the equilibrium is full pooling.

Equilibrium Picture



LCSE Theorem

- Will the Least Cost Separating Equilibrium (LCSE) survive?

Theorem

The LCSE is the unique D1 equilibrium iff

$$\frac{C_L}{C_H} \geq E[R(g|\bar{x}(v_L))|L]$$

- Condition is easier to satisfy as
  - the cost advantage, $\frac{C_L}{C_H}$, increases.
  - informativeness, $E[R(g|\bar{x}(v_L))|L]$, decreases.
- LCSE never survives if $C_H = C_L$.

Full Information Outcome

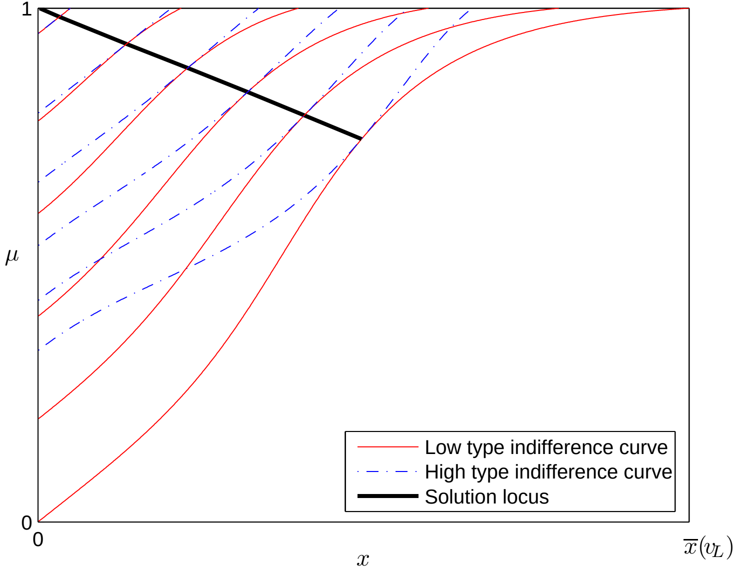
- If the receivers "knew" the sender was a high type, he would not need to signal.
- In the standard model, for any $\mu_0 < 1$, the unique D1 equilibrium is the LCSE.
- With grades, the sender chooses a less expensive signal as μ_0 increases.
- Can the equilibrium converge to the full information equilibrium of full pooling at $x = 0$ as $\mu_0 \to 1$?

Convergence Condition

- For the LCSE equilibrium to survive, we need the solution locus to coincide with the upper boundary.
- For Convergence we need exactly the opposite:

  The equilibrium converges to full pooling at $x = 0$
  iff the solution locus stays below $\mu = 1$ for $x > 0$.

Convergence Picture



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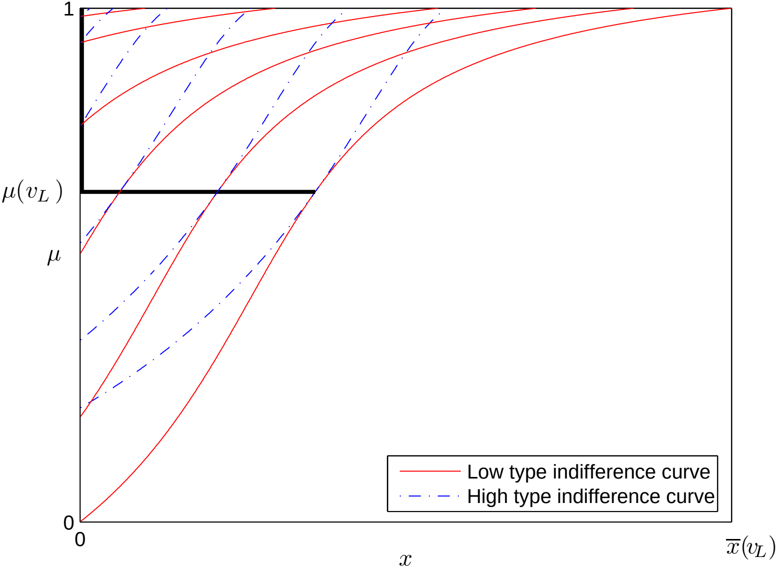
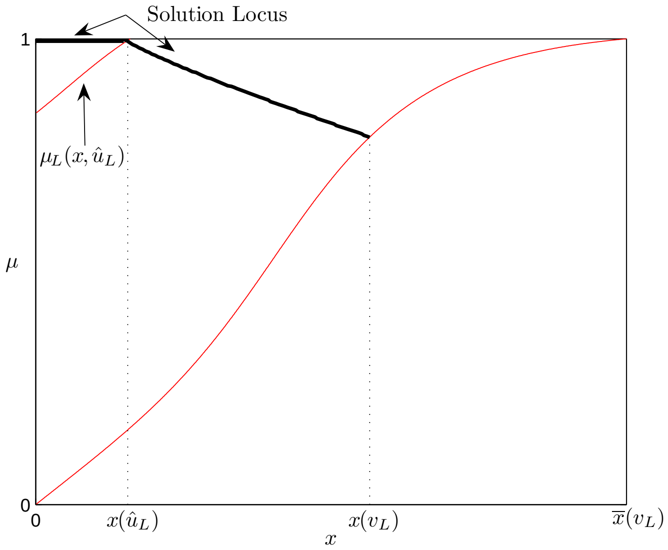


Convergence Picture



Convergence Theorem

Theorem

If $E[R(g|x)|L]$ *is increasing (constant) at* $x = 0$*:*

As $\mu_0 \to 1$*, the set of equilibria converges to the full pooling equilibrium at* $x = 0$ *if and only if*

$$E[R(g|0)|L] \geq (>) \frac{C_L}{C_H}$$

.

Corollary

If $C_L = C_H$ *the set of equilibria converges to the full pooling equilibrium at* $x = 0$ *as* $\mu_0 \to 1$*.*

Future Work

- The market for *graders*.
- Dynamic Criticism: If x takes time, receivers may choose to preemptively offer wages to the sender.
- Gradeless model breaks down (no signaling), when receivers are allowed to do so (Swinkels 1999).
- We investigate the analog of the dynamic signaling game, by allowing grades to accumulate gradually over time.

  When grades accumulate at a sufficient rate, trade is not always immediate—signaling will occur.

Conclusion

- We have characterized the precise interaction between Signaling and Grades.
  - The high type's relative, not absolute, advantages matter.
- When grades are informative, types can no longer be distinguished by actions alone.
- Grades solve the "failure-to-pool-even-with-high-prior" problem.

  Explain why senders with lower priors must expend more resources signaling.